Gamma-ray spectroscopy of deformed states in light nuclei and cluster emission

by

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Zusammenfassung

Im Rahmen dieser Arbeit wurde nach $\gamma$-Übergängen möglicher Cluster-Strukturen in leichten Kernen gesucht. Dazu wurden Experimente mit dem $\gamma$-Detektorball GASP am Laboratori Nazionali di Legnaro (LNL) in Italien durchgeführt. Um eine bessere Kannaltrennung und Selektivität zu erreichen wurde zusätzlich der Detektorball ISIS (zum Nachweis leichter Teilchen) verwendet. Mit der Hilfe von $\gamma$-Teichen-Koinzidenzen wurden detaillierte Niveau-Schemata für die Übergänge unter der Teilchenschwelle in $^{10}$Be extrahiert. Außerdem wurde die Bevölkerung der $\gamma$-Übergänge durch Direkte- und Compoundkern-Reaktionen für die Neon-Isotope studiert. Das beobachtete $\gamma$-Spektrum für $^{21}$Ne wurde als starker Hinweis auf eine reflexionsasymetrische Struktur gedeutet. Neue Spinzuordnungen, die einer DCO (Directional Correlations de-exciting Oriented states) Analyse folgen, sind für einige der Niveaus in $^{21}$Ne getroffen worden. Neue Übergänge in $^{22}$Ne wurden gefunden. Durch Vergleich der von uns gemessenen Daten mit theoretischen Vorhersagen konnten zusätzlich vorläufige Spinzuordnungen für $^{23}$Ne extrahiert werden.

Eine quantitative Analyse der Emission von $^8$Be und $^{12}$C* wurde durchgeführt und aus den Energiespektren wurden Informationen über das Verhältnis zwischen den gleichzeitig eintreffenden unkorrelierten $\alpha$-Teilchen sowie die den realen $^8$Be-Clustern extrahiert. Zusätzlich wurden die Unterschiede zwischen den registrierten $\gamma$-Spektren in Koinzidenz mit zwei $\alpha$-Teilchen in unterschiedlichen Detektoren, und den $\gamma$-Spektren in Koinzidenz mit den \textquoteleft$^8$Be\textquoteright Ereignisse analysiert.

Es wurde experimentell nachgewiesen, daß die Restkerne nach der Cluster-Emission in einem höheren Anregungszustand sind, als die gleiche Kerne, die aus der aufeinanderfolgenden Emission der $\alpha$-Teilchen entstanden sind. Deshalb ist bei Restkerne, die aus der Cluster-Emission hergegangen sind, die Emission weitere leichte Teilchen bevorzugt. Dieses Phänomen ist diskutiert und eine mögliche Erklärung wurde gegeben.
Abstract

Gamma-ray decays from possible nuclear cluster structures in light deformed nuclei have been investigated using the GASP array of high purity germanium detectors. In order to achieve the required experimental sensitivity, a special device was used, namely a highly efficient array of silicon-detector telescopes for the detection of charged particles. Using $\gamma$-particle coincidences, a detailed level scheme for the $\gamma$-ray transitions in $^{10}$Be beneath the particle threshold was obtained and new $\gamma$-ray transitions identified. Furthermore, the $\gamma$-ray populations in the direct and compound reaction mechanisms were studied for neon-isotopes. The $\gamma$-ray spectra obtained for $^{21}$Ne were interpreted as indicating a reflection asymmetric structure. New spin assignments have been made for some of the levels in $^{21}$Ne following a DCO (Directional Correlations de-exciting Oriented states) analysis. New transitions in $^{22}$Ne were found. In addition, tentative spin assignments for $^{23}$Ne were extracted after comparing the current experimental data with theoretical predictions.

A quantitative analysis of the emission of $^{8}$Be and $^{12}$C* clusters was made and information about the ratio between the coincident uncorrelated $\alpha$-particles as well as the real cluster events was extracted from the energy spectra. In addition, the differences between the $\gamma$-ray spectra in coincidence with two $\alpha$-particles registered in different detectors and the $^{8}$Be’ events were analysed. The results show the enhanced sequential emission of $\alpha$-particles. Here, it has been experimentally observed that the residual nuclei, after cluster emission, are in a state of higher excitation energy than the same compound nuclei following the sequential emission of $\alpha$-particles. This phenomenon is discussed and an explanation proposed.
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To my family, Grandma and Carl
“The only thing to do with good advice is to pass it on. It is never of any use to oneself”

by Oscar Wilde.
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Chapter 1

Deformation and clustering in light nuclei

1.1 Introduction

Questions relating to how states of matter arise and what the building blocks are from which they are made will never stop to interest people. After the discovery of the atom and the nucleus physicists started to investigate atomic nuclei and their properties. During this period many fundamental discoveries were made such as the observation of new elementary particles involved in radioactive decay. Today, a large part of the Segré chart of the nuclides is well known. Numerous theoretical nuclear models have been developed and accepted or rejected based on experimental work during this period. In addition, modern and more powerful accelerator facilities have been developed.

One of the interesting problems discussed in the last 30 years is that of cluster structure in nuclei and the existence of so called ‘nuclear molecules’. Different projects and people have been involved in these investigations. Now, after twenty years of scepticism from a large part of the physics community it has been proven that such structures exist. Still, there is a lot of work which has to be done towards understanding such structures, especially for light nuclei. This work is a small
piece from the big puzzle of ‘nuclear structure’, but putting such pieces together will eventually make it possible to see the whole picture.

In light nuclei, the nucleons have been observed to cluster together forming sub-structures within the atomic nucleus, for states where the nucleons are only just bound together. This fact is well expressed in the Ikeda Diagram (see Figs. 1.1 and 1.2). In neutron-rich nuclei low-lying states, close to the threshold for neutron emission, show a pronounced α-particle cluster structure. These are states with large prolate deformations. For the neutrons outside the tightly bound core (so called ‘valence neutrons’) a concept based on molecular orbitals can be used to describe their behaviour.

Figure 1.1: Ikeda Diagram [Ike68, Hor72] showing cluster states in $N = Z$ nuclei and their thresholds (in MeV) for the decomposition into clusters.
Figure 1.2: Extended Ikeda Diagram, which schematically illustrates the molecular shape isomers based on $\alpha$ (green) and $^{16}\text{O}$ (blue) clusters plus some covalently bound neutrons (red) in neutron-rich light nuclei. Numbers under the configurations indicate the thresholds (in MeV) for decaying into the subunits [vO01].
The physics of molecular orbitals for nucleons (mostly neutrons) has been developed and successfully applied in the last decades for the description of transfer processes in heavy-ion reactions at low energies [vO70, Ima87, Bis88, vO96b, Spa00]. In these models weakly bound neutrons and strongly bound cores are used. The valence neutrons move in the field of two clusters and some aspects from atomic molecular physics can be applied to such systems.

Thus, conditions for the formation of stable or quasi-stationary molecular states can be formulated [vO01]:

- strongly bound cores.
- weakly attractive core-core potential which, in addition, becomes repulsive at small distances.
- weakly bound single-particle orbitals of valence neutrons in order to guarantee large amplitudes of the wave functions at larger distances in the overlap region.
- large transfer probability, which is typically reached if the valence states are in the resonance or in a quasi-resonance matching condition between the two states of the separated centres.

Covalent binding between $\alpha$-particles due to valence neutrons produces particular structures in light neutron-rich nuclei, like in the beryllium isotopes, namely long-lived states with a two-centre structure. The long lifetime arises because of the dramatic change in the shape of the state needed to decay to lower-lying ‘normal’ states. Such unusual arrangements of nucleons also give rise to reflection asymmetric shapes, like in atomic molecules, a signature of which are deformed bands as parity doublets. These cases are well known in atomic physics and discussed by Herzberg [Her50]. For such structures using the two centre shell-model a correlation diagram for all nucleons (see Fig. 1.3) can be calculated [vO70, Sch71]. The molecular orbits merge at small distances with the Nilsson orbits (see Fig. 1.5) of the deformed compound nucleus. The molecular orbitals are classified according to the well known quantum numbers of molecular valence
Figure 1.3: Correlation diagram for molecular orbitals in a two-centre shell-model picture. The molecular orbitals are labeled by their quantum numbers (see text). The distance between the two centres is denoted by $r$.

states: the $K$-quantum number for the projection of the total angular momentum on the nuclear deformation axis, and the $\sigma$ and $\pi$ orbitals for the $m=0$ and $m=1$ projections of the orbital angular momentum $l$ respectively. In addition to the parity, the gerade (even), $g$, and ungerade (odd), $u$, symmetry appears for the case of two molecular cores.

### 1.2 Aims and techniques of the experiments

Some spectroscopic properties that are important for molecular structures are:

- large transition probabilities for $\gamma$-decay or/and a large probability for cluster emission.
- the reaction mechanism in which the nucleus is produced.
• large moment of inertia.
• rotational bands with intense intra-band $\gamma$-ray transitions.

The study of the atomic nucleus using particle and $\gamma$-ray spectroscopy and particle-$\gamma$ coincidence techniques has several advantages. From one side it gives the opportunity to investigate the electromagnetic transitions below and across the particle-emission threshold, and from the other side the study of the $\gamma$-ray decay scheme in these nuclei helps to establish a clear signature of collectivity - namely rotational structures and in particular parity doublet structures characteristic of reflection asymmetric two-centre structures. Using $\gamma$-ray spectroscopy it is possible to find the connection between previously known states in these isotopes and furthermore to observe new behaviour.

The main goal of this work is the study of the level schemes of beryllium and neon isotopes in order to established their structures as related to the molecular model. The emission of clusters in compound nuclear reactions, observed in these studies, is another important part of the work presented here.

1.3 Deformed shell model (Nilsson model)

The spherical shell model [May49] can explain many features of spherical nuclei, but needs modifying to describe nuclei with many nucleons outside a closed shell. The residual interaction between these many valence nucleons may be more simply described by a deformed potential.

For nuclear rotation to be observable, the nuclei have to be non-spherical, so that they have a preferred axis. For deformed nuclei, assuming a constant nuclear volume (i.e. incompressibility), the nuclear radius can be described by:

$$ R(\theta, \phi) = R_{av} \left[ 1 + \sum_{\lambda=2}^{\infty} \alpha_{\lambda \mu} Y_{\lambda \mu}(\theta, \phi) \right] $$

(1.1)

where $\alpha_{\lambda \mu}$ are the coefficients of the spherical harmonics $Y_{\lambda \mu}(\theta, \phi)$ [Eis70]. The $\lambda=1$ terms are normally excluded from the sum as these correspond to a transla-
tion of the centre of mass. The indices $\lambda$ and $\mu$, determine the surface coordinates as a function of $\theta$ and $\phi$, respectively. For example,

$$R(\theta, \phi) = R_{av} [1 + \beta_2 Y_{20}(\theta, \phi)]$$

is independent of $\phi$. This means that such nuclei are axially symmetric and can be either prolate or oblate (see Fig.1.4). The deformation parameter $\beta_2 (= \alpha_{20})$, can be related to the axes of the spheroid by:

$$\beta_2 = \frac{4}{3} \sqrt{\frac{\pi}{5}} \frac{\Delta R}{R_{av}}$$

in which the average radius is, $R_{av} = R_0 A^{1/3}$, and $\Delta R$ is the difference between the semi-major and semi-minor axes.

Figure 1.4: Currently observed nuclear shapes [Luc01]. The different shapes can be parametrised by spherical harmonic functions, where $\lambda$ characterises the different orders of the corresponding distributions.

The larger the value of $\beta_2$ the more deformed the nucleus. Positive and negative $\beta_2$ values correspond to prolate and oblate shapes respectively.

In some circumstances the quadrupole deformation parameters $\varepsilon_2$ and $\delta$ are used. These are related to $\beta_2$ by the Equations (1.4), taken from [Fir96].

$$\delta = \frac{\Delta R}{R_{r.m.s.}}$$
\[ \varepsilon_2 = \delta + \frac{1}{6} \delta^2 + \frac{5}{18} \delta^3 + \frac{37}{216} \delta^4 \ldots \]
\[ \beta_2 = \sqrt{\frac{\pi}{5}} \left[ \frac{4}{3} \varepsilon_2 + \frac{4}{9} \varepsilon_2^2 + \frac{4}{27} \varepsilon_2^3 + \frac{4}{81} \varepsilon_2^4 \ldots \right] \] (1.4)

Higher order axially symmetric effects have also been observed in nuclei, such as hexadecapole deformation quantified by \( \beta_4 \) (or \( \varepsilon_4 \)).

The shape parameters introduced so far all describe axially symmetric nuclear shapes, but quadrupole (\( \lambda = 2 \)) deformations can give rise to asymmetric shapes. These triaxial distortions are governed by the \( \gamma \) shape degree of freedom, and this describes a stretching/squashing effect at right angles to the major nuclear axis. Gamma is measured in degrees, where \( \gamma = 0^\circ \) and \( \gamma = 60^\circ \) correspond to prolate and oblate shapes respectively. Completely triaxial shapes have \( \gamma = 30^\circ \).

The model that describes axially symmetric nuclei is called the Deformed Shell Model. In this model the Schrödinger equation is solved using the potential that describes, as closely as possible, the actual shape of the nucleus. Another result of the deformation is that the orbital angular momentum, \( l \), and the intrinsic spin, \( s \), are no longer good quantum numbers and thus, states with different \( l \)-values, but the same parity can mix. The energy of the states now depends on the component of the single-particle angular momentum (\( j \)) along the symmetry axis, which is denoted by \( \Omega \). For each orbital with angular momentum \( j \), there are \( 2j + 1 \) values of \( \Omega \) (\( = m_j \) in the absence of other couplings). However, levels with \( +\Omega \) and \( -\Omega \) have the same energy due to the reflection symmetry of axially symmetric nuclei, so that each state is now doubly degenerate, \( i.e. \) two particles can be placed in each state. For example the \( f_{7/2} \) orbital can have \( |\Omega| \) equal to 7/2, 5/2, 3/2 and 1/2. The ordering of these \( \Omega \) levels depends on the particular shape of the nucleus since the lowest in energy is the orbital which interacts (overlaps) the most with the nuclear core. For prolate shaped nuclei the states with the lowest \( \Omega \) values are the most tightly bound, whereas for oblate shaped nuclei, the states with the highest \( \Omega \) occur lowest in energy. Such deformed shell model calculations were first performed in 1955 by Nilsson [Nil55] with an anisotropic harmonic
Figure 1.5: Nilsson orbitals for nuclei up to $Z=N=50$ taken from [Fir96]. The level ordering is given as a function of the quadrupole deformation parameter, $\varepsilon_2$. Dashed lines indicate negative parity and solid lines indicate positive parity. Positive and negative values of $\varepsilon_2$ correspond to prolate and oblate shapes respectively. See text for more information.
oscillator potential and the calculated states (called Nilsson orbitals) are labelled by \( \Omega[Nn_z\Lambda] \) (see Fig.1.5), where \( N \) is the total oscillator shell quantum number and determines the parity, given by \((-1)^N\). Lambda (\( \Lambda \)) is the projection of the particle orbital angular momentum, \( l \), on the nuclear symmetry axis, and \( n_z \) is the number of oscillator shell quanta along the direction of the symmetry axis.
Chapter 2

Experimental method

2.1 Reaction mechanism

Compound nuclear reactions can populate continuum states of nuclear many-body systems. These states have high excitation energies, and therefore such reactions tend to involve complicated excitations involving many degrees of freedom. Reactions with these characteristics proceed further by the compound-nucleus (CN) mechanism. However, it may sometimes happen that only a few degrees of freedom are excited and that the other degrees of freedom of the many-body system effectively remain passive. Reactions that possess this simplicity are said to proceed by the direct-reaction (DR) mechanism. Cross-sections from both mechanisms are found experimentally and it is necessary to build theories that are adapted to both the CN and DR types. Often the reaction mechanism is analysed using two quite different theoretical techniques, with the DR analysis stressing detailed solutions of the Schrödinger equation and the CN analysis concentrating on statistical properties of the nuclear system. However, it is clear that both CN and DR types can be present simultaneously and that the two aspects of any given nuclear reaction must be considered. This is particularly true for reactions between light nuclei. A short description of both reaction mechanisms is given below.
2.1.1 Compound nuclear reactions

The idea of compound nucleus formation was first suggested by Niels Bohr in 1936 [Boh36]. In such a reaction $A_1 + A_2 \rightarrow C^*$, the target and the projectile fuse together, forming a compound nucleus, $C^*$. In this model several criteria must be satisfied:

- the projectile must have an energy large enough to overcome the Coulomb barrier ($E_{CB}$):
  \[ E_{CB}[MeV] = \frac{1.44Z_1Z_2}{1.16(A_1^{1/3} + A_2^{1/3} + 2)} \] (2.1)
  where $Z_1$ and $A_1$ are the atomic number and mass of the projectile respectively and $Z_2$ and $A_2$ are the corresponding values for the target nucleus.

- the angular momentum transfer should be small so that the centrifugal repulsion caused by the rotation of the nucleus does not overcome the short-range attraction of the nuclear force.

- all degrees of freedom of the compound system must be equally populated.

The subsequent decay is governed by the phase space of the individual channels $X; A_1 + A_2 \rightarrow C^* \rightarrow X + x$.

When formed, the compound nucleus will be in a state of high excitation energy. This excitation energy can be lost from the compound nucleus in several ways, as shown in Fig. 2.1. At high angular momenta the most probable way is fission. This is due to the fact that the nucleus is no longer stable against the centrifugal force caused by the rotation, which means that the attraction of the nuclear force can be overcome. If the compound nucleus does not undergo fission, the main part of the excitation energy is removed by particle emission (neutrons, protons and $\alpha$-particles). The evaporation of charged particles (protons and $\alpha$-particles) is normally suppressed by the Coulomb barrier. The evaporation of particles carries away a large amount of excitation energy (10-20 MeV) [Mor83], but a small amount of angular momentum. Thus, the evaporation of particles leaves the compound nucleus in a state of high angular momentum, and gives a
Figure 2.1: Diagram of the excitation energy of the compound nucleus, versus spin $I$. The scheme shows the decay of the compound nucleus via particle and $\gamma$-ray emission.

steep descent towards the ground state.

When the excitation energy is below the particle threshold the nucleus continues to de-excite via the emission of statistical $\gamma$-rays due to the very high density of states. These are usually dipole transitions (E1), carrying away energy, but again very little angular momentum. These $\gamma$-rays are normally not resolved in a $\gamma$-ray spectroscopy experiment. After the nucleus is cooled it reaches the yrast (which means weary in Swedish) region. The yrast line defines the states with the largest spin value, $I$, for a given excitation energy. The $\gamma$-ray emission continues via discrete $\gamma$-ray transitions (e.g. E2 decays), carrying away more angular momentum until the ground state is reached. These yrast-like transitions are
the most important from a spectroscopic point of view since they carry enough intensity to be isolated in a γ-ray spectrum and from them information about the studied nuclei can be extracted.

2.1.2 Direct reactions

In contrast to the compound nucleus reaction mechanism direct reactions involve direct passage from the initial to the final state, without the formation of any intermediate state.

Stripping reactions

The theory of stripping reactions was first outlined, for low energies and without discussion of angular momentum, by Oppenheimer and Philips [Opp35]. More detailed treatments, concerned especially with nuclear states using angular distributions to assign values of angular momentum to the states formed in stripping reaction at higher energies, have been published by Butler [But51] and by Bhattia [Bha52].

(d,n) reactions

The best known direct reactions are stripping reactions, in which, for example, an incident deuteron is stripped of one of its nucleons, which enters the target nucleus, leaving the other to continue an independent life outside (see Fig. 2.2).

The details of such reactions are determined from the properties of the target nucleus, which fixes the energy and the angular momentum with which the proton must enter it. The reaction is not usually observed unless these quantities are correct for forming a well-defined state in the final nucleus.

The orbital angular momentum, \( l \), that the proton needs to take it into the target nucleus is determined by the angular momenta and parities of the initial and final nuclear states. Conservation of angular momentum limits \( l \) to one of two
consecutive integer values, since the difference between the total angular momenta of the initial and final states must be equal to $l + 1/2$ or $l - 1/2$ (according to whether the proton has its spin parallel or antiparallel to $l$). The conservation of parity (see [Hug71, Per71]) fixes which of the two $l$-values will be effective. If the parities of the initial and final states are the same, $l$ must be even and if the parities of the states are different $l$ must be odd.

The kinetic energy of the neutron is determined by how much of the incident energy of the deuteron is taken into the target nucleus by the proton. In fact it serves to identify the final state produced, by the $Q$-value of the reaction as a whole.

The basic theory of stripping reactions, as outlined by Butler and Hittmair [But57] starts with three approximations, namely that the following interactions may be neglected:

- interaction of the continuing particle (the neutron in a $(d,n)$ reaction and the proton in $(d,p)$ reactions) with the initial nucleus;
- interaction of the neutron with the proton after one of them has entered the target nucleus;
- interaction of the target nucleus with the deuteron as a whole, leading to elastic scattering.

Under these approximations, which are valid for incident deuteron energies
greater than a few million electronvolts, the problem can be written down as an initial wave function describing the target nucleus interacting with an incident plane wave of deuterons, including the internal wave function of the deuteron, and a final wave function obtained. The latter must describe the captured proton inside the nucleus as well as the neutron departing in partial waves, with $l$ determined by the angular momentum with which the proton entered the nucleus. Capture of the protons with a given $l$ thus causes the neutron to depart in a definite set of partial waves, and hence with a characteristic angular distribution. For example, capture with $l = 1$ in the reaction $^{11}\text{B}(d,n)^{12}\text{C}$ at 8 MeV gives a first peak in the angular distribution of neutrons at about 20°.

**The (d,p) reactions**

If it is the neutron that is captured, while the proton remains outside the nucleus, the process is a $(d,p)$ reaction which may be described by a theory differing only marginally from that outlined above. The main difference is that allowance must be made for the Coulomb interaction of the departing particle and the final nucleus. It results in no qualitative changes, but the peaks in the angular distributions are shifted outwards to slightly greater angles. On the other hand, treatment of (d,p) reactions is simpler in that the interaction of the captured particle with the target nucleus contains no Coulomb component.

**Pick-up reactions**

With incident energies less than about 5 MeV $(n,d)$ and $(p,d)$ reactions are likely to proceed mainly through compound nucleus formation. At higher energies the inverse of a stripping reaction (a pick-up reaction) may be important. If the target nucleus has an appreciable probability of being found in a state equivalent to a free neutron and a residual nucleus in a state which could exist alone, a passing proton may pick-up the neutron and form a deuteron. For this to happen, the neutron must be in a state which allows it, when free, to have a momentum close
to that of the incident proton; if there is too much discrepancy, the interaction will be too weak to form the deuteron.

2.2 Gamma-ray detectors and charged-particle detectors

The study of clustering in nuclei has been carried out over several decades using transfer reactions combined with particle spectroscopy. Studying these nuclei with \( \gamma \)-ray spectroscopy is of interest because in this case the rotational structure of deformed states can be distinguished through the properties of the \( \gamma \)-decay. Since the states of interest are populated with very low probabilities, special techniques are required. During the last decade the development of new arrays of germanium detectors with large efficiencies has opened up many new possibilities to study nuclear states populated with very small cross-sections with respect to the total reaction cross section.

2.2.1 GASP (GAmma-ray SPectrometer)

The GASP array [Baz92] at the Laboratori Nazionali di Legnaro, Italy, consists of 40 Compton-suppressed, hyper-pure, high-efficiency, n-type germanium detectors (HpGe) which are placed 27 cm from the target (see Fig. 2.3) and cover a total solid angle of 10\% of \( 4\pi \). This corresponds to a total absolute photo-peak efficiency of \( \approx 3\% \) at a \( \gamma \)-ray energy of 1332 keV. These detector systems have been designed to observe, with high efficiency, \( \gamma \)-ray cascades with high multiplicities from compound reactions, following the evaporation of a few particles (n, p and \( \alpha \)).

The high selectivity of large arrays of high-purity germanium detectors like GASP is not sufficient by itself to select weakly populated light deformed nuclei. Higher selectivity can be obtained by combining the germanium array with an-
other special ancillary detector which allows triggering on the particle emission of interest. Such a detector is the silicon-ball, which has been developed for the in-beam study of nuclei requiring a high degree of channel selectivity.

![Figure 2.3: A picture of the $\gamma$-ray detector array GASP. The second half is moved away. In the centre is the silicon detector ball ISIS.](image)

### 2.2.2 ISIS (Italian SiIlicon Sphere)

The ISIS (see Fig. 2.4) consists of 40 silicon $\Delta$E-E telescopes positioned in a compact (16 cm diameter) sphere. Each telescope is composed of two silicon detectors: a thin (130 $\mu$m) detector facing the target, behind which a second thick (1 mm) detector is placed. The ISIS array was developed to serve as an ancillary device for the GASP $\gamma$-ray spectrometer [Far97] and its geometry closely resembles that of GASP, with one telescope in front of every germanium detector. This geometrical configuration has been chosen to minimise the $\gamma$-ray absorption and scattering which would reduce the detection efficiency and the peak-to-total
The energy loss of charged particles in matter can be described using the simplified Bethe-Bloch equation \cite{Kno99}:

\[
\frac{dE}{dx} \propto \frac{mZ^2}{E},
\]  

(2.2)

where \( m \) and \( Z \) refer to the mass and atomic number of the incoming particle respectively. Plotting the energy signal of the first (thin) detector versus the signal of the second (thick) detector, the events for each different value of the parameter \( mZ^2 \) will be separated into a distinct ‘banana’ shaped region. This allows a clean selection to be made on different types of emitted particles.


2.3 Data analysis

2.3.1 Structure of the data and channel selection

Data are written to magnetic tapes in list mode, which means that every time a new event occurs, the signals from each detector that fires are digitised and written out, together with a number identifying the detector. For the data discussed here, the master trigger is chosen to be GASP and the occurrence of a new event was determined by the condition that at least two Ge-detectors fired in coincidence, within a fixed time window (≈180 ns).

The tapes were read using the GSORT program [Baz97]. This is a general purpose sorting program, which accepts the pre-defined commands written in a common file and user-written subroutines can be implemented. Using this program, raw data can be rearranged in one-, two- and three-dimensional histograms called spectra, matrices and cubes respectively. In the subsequent analysis the histograms were handled using the TRACKN [Baz97] and CMAT [Baz97] programs of the GASP data analysis package.

Since the cross section for the nuclei of interest are smaller than those of many other competing reaction channels it is difficult or even impossible to study their structure using only γ-ray detectors.

2.3.2 Calibrations

When using an array of germanium detectors it is important to have the possibility of summing the data from different detectors. An absolute energy calibration is needed for all individual detectors to deduce the energies of the newly observed γ-ray transitions. For this reason, data with standard radioactive sources of $^{152}$Eu and $^{56}$Co were collected. Energy calibration coefficients were obtained by fitting the source spectra to produce aligned one dimensional spectra (without further conditions), called projections, for each detector and for each data file.
The projections were subsequently analysed to produce the alignment coefficients matching each run to the reference run (the last in-beam data file was chosen as the reference since this was the closest in time to the runs taken with radioactive sources). This minimised the risk of drifts in the electronics between the experiment and the source calibration runs.

In order to further simplify the analysis an alignment was performed for the silicon detectors. In our case they were simply used to identify the incoming particles, without the need for an absolute energy calibration.

Having calibrated and aligned the energy spectra for the germanium and silicon detectors respectively, the data were sorted into matrices and cubes on disk. A matrix here refers to a two-dimensional histogram where each axis corresponds to a defined quantity, such as the signal of a particular class of detectors or a combination of signals. For example, a $\gamma$-$\gamma$ matrix is generated by histogramming the energy signal of any germanium detector on one axis and on the other axis the energy signal of every other germanium detector that fired in coincidence with the first. Various matrices and cubes were produced with different conditions on the $\gamma$-ray energies and time and on the particle multiplicity.

One has to take into account that the photopeak efficiency of a detector varies with the photon energy, consequently the area of the photopeak in the spectrum will depend both on the absolute number of emitted photons and on the photopeak efficiency at a given energy. One problem encountered when determining the correct intensities of weak transitions is that they are not directly visible in the projection of a matrix. They can only be seen by gating on another transition. Therefore, in order to deduce their intensities, it is necessary to correct for the efficiency at the gating energy.

The relative detector efficiency calibration is determined by analysing the spectra taken with standard radioactive sources of $^{152}$Eu and $^{56}$Co, for which the intensity ratios of the transitions are well known [htt02]. The two sources used here cover the energy range between 40 keV and 3500 keV: at low energies points
on the efficiency curve were provided by the X-rays and γ-rays coming from the 
$^{152}\text{Eu}$ source, the high energy points were obtained from the $^{56}\text{Co}$ source lines.

To obtain an efficiency calibration for any array of germanium detectors with a similar efficiency response, the spectra of all the Ge-detectors are summed. The resulting spectrum is analysed to find the area of the known peaks, which can be considered as discrete efficiency points. These discrete points can be subsequently fitted with a semi-empirical expression (see Fig. 2.5).

![Efficiency curve for the Ge-detectors obtained with the standard radioactive sources $^{152}\text{Eu}$ and $^{56}\text{Co}$.](image)

Figure 2.5: Efficiency curve for the Ge-detectors obtained with the standard radioactive sources $^{152}\text{Eu}$ and $^{56}\text{Co}$.

For lines which are out of range of known points on the efficiency curve, an extrapolation using the known efficiency function must be performed, leading to a larger uncertainty in the final transition intensities. For example, for energies higher than 4000 keV (see Fig. 2.5) one should keep in mind the uncertainty arising from the extrapolation.
2.3.3 Kinematical corrections and Doppler broadening

The intrinsic resolution of a HPGe crystal is approximately 2-2.5 keV for 1332 keV γ-rays. The performance of γ-ray spectrometers is often limited by the Doppler broadening of the lines due to the fact that photons are emitted while the recoiling nucleus is in-flight. The well known expression for the Doppler shift gives the observed energy $E_\gamma$:

$$E_\gamma = E_0^\gamma \frac{\sqrt{1 - \beta}}{1 - \beta \cos \theta} \approx E_0^\gamma (1 + \beta \cos \theta),$$

(2.3)

where $E_0^\gamma$ is the energy of the emitted γ-ray if the nucleus is stopped, $\beta$ is the recoil speed, expressed as a fraction of the speed of light and $\theta$ is the angle between the recoil velocity and the direction of observation for the γ-ray. Differentiating Equation 2.3, yields the broadening of the lines, to first order, induced by the recoil of the residual nuclei:

$$\frac{\Delta E_\gamma}{E_\gamma} = \cos^2 \theta \cdot (\Delta \beta)^2 + (\beta \sin \theta)^2 \cdot (\Delta \theta)^2$$

(2.4)

Thus, it can be seen that the loss of resolution comes from two components, originating from the dispersion in the recoil velocity and in the angle between the direction of the γ-ray and the direction of the recoiling compound nucleus. The final effect for typical recoil velocities in fusion-evaporation reactions with $\beta \sim 1\rightarrow 4\%$ of the velocity of light can easily be of the same order of magnitude (or larger) than the intrinsic resolution of the germanium crystal.

Assuming that the detector covers an infinitesimal solid angle the effect will be that all of the detected photons would be shifted by the same amount with respect to the energies of the emitted photons. Since real detectors cover a finite solid angle, photons will interact within a finite angular range $[\theta - \Delta \theta, \theta + \Delta \theta]$ and the effect will be that the peaks will be broadened by a finite amount, which can be estimated to first order as:
\[ |\Delta E_{\gamma}^0| \approx \left| 2\Delta\theta \frac{\partial E_{\gamma}}{\partial \theta} \right| = 2\Delta E_{\gamma}^0 \beta |\sin\theta| \] (2.5)

where \( \Delta\theta \) is half of the opening angle of the detectors. For GASP \( \Delta\theta \approx 14.5^\circ \)

Other effects inducing a velocity dispersion on the recoiling nuclei contribute to the Doppler broadening of the peaks, such as the interaction with the atoms in the target and the evaporation of light particles.

In fusion-evaporation reactions the momentum is conserved, which means that the momentum of the recoiling nucleus, \( \vec{p}_R \):

\[ \vec{p}_R = \vec{p}_{CN} - \sum_{i=1}^{n} \vec{p}_i \] (2.6)

where \( n \) particles are evaporated from the compound nucleus. This implies that,

\[ E_{\gamma} = E_{\gamma}^0 \left( 1 + \frac{\vec{v}_R \cdot \vec{d}_{\gamma}}{c} \right) \] (2.7)

where \( \vec{d}_{\gamma} \) is the unit vector in the direction of the emitted \( \gamma \)-ray. The expression of the observed energy, \( E_{\gamma} \), taking into account the emission of light particles is:

\[ E_{\gamma} = E_{\gamma}^0 \left( 1 + \frac{m_{CN} \vec{v}_{CN} \cdot \vec{d}_{\gamma}}{m_R c} - \sum_{i=1}^{n} \frac{m_i \vec{v}_i \cdot \vec{d}_{\gamma}}{m_R c} \right) \] (2.8)

By detecting the emission vector of the evaporated light charged particles it is possible to perform an event by event reconstruction of the velocity of the recoiling nucleus, thus allowing a more precise Doppler shift correction \[\text{[Sew94]}\]. The effect of this reconstruction is two fold, improving both the effective resolution of the detectors and the peak-to-total ratio. This is equivalent to an increase in the resolving power of the \( \gamma \)-ray spectrometer, which when combined with the selectivity offered by the coincidences with charged particles, enables the study of nuclei populated with small cross sections.

From the experimental data obtained with ISIS it is not possible to get exact energy measurements for the light charged particles because:

- the energy loss from the particles in the absorbers is \textit{a priori} not known.
• the E-detectors are too thin to stop the high energy protons fully at forward angles.

Therefore, instead of an exact energy a mean value (depending on the angles) is used and optimised with successive approximations. Despite its empirical nature, the method produces good results.
Chapter 3

Spectroscopy of beryllium isotopes

3.1 Theoretical considerations for the Be-isotopes

3.1.1 Cluster model

It is well known that in the $N = Z$ region a well developed cluster structure appears for lithium and beryllium isotopes. Typical examples of this are the $\alpha-\alpha$ structure of $^8\text{Be}$ and the $\alpha-t$ structure of $^7\text{Li}$ [Ike80]. Many theoretical studies using the cluster model have been successfully made both for structure problems and for nuclear reaction problems. However, a small part of these theoretical investigations have calculated the structure of these nuclei without assuming the existence of clusters. Below, the method of antisymmetrised molecular dynamics is described briefly. This is a microscopic model which is free of any assumption about the existence of clustering and starts from the nucleon-nucleon interaction alone.
3.1.2 Antisymmetrised Molecular Dynamics

This theoretical method was developed by Akira Ono et al. for the study of nuclear reactions [Ono92a, Ono92b, Ono93b, Ono93a, Ono95]. With Kanada En’yo this work was extended and applied very successfully to the study of nuclear structures [KE95b, KE95a, Dot97, KE98a, KE99]. The method of Antisymmetrised Molecular Dynamics (AMD) has already proved to be a powerful theoretical tool for the study of the ground states as well as excited states in light nuclei.

The simplest version of AMD for the study of nuclear structure

The wave function of a system can be written as a linear combination of AMD wave functions:

\[ \Phi = c \Phi_{AMD} + c' \Phi'_{AMD} + \ldots \]  

(3.1)

where the wave function of a nucleus with mass number \( A \) is a Slater determinant of Gaussian wave packets:

\[ \Phi_{AMD}(Z) = \frac{1}{\sqrt{A!}} A \{ \varphi_1, \varphi_2, \ldots, \varphi_A \}, \]

\[ \varphi_i = \phi_{X_i} \mathcal{X}_i \tau_i : \begin{cases} \Phi_{X_i}(r_j) \propto \exp \left[ -\nu \left( r_j - \frac{X_i}{\sqrt{\nu}} \right)^2 \right], \\ \mathcal{X}_i = \begin{pmatrix} \frac{1}{2} + \xi_1 \\ \frac{1}{2} - \xi_1 \end{pmatrix}, \end{cases} \]

where the \( i \)-th single-particle wave function \( \varphi_i \) is a product of the spatial wave function \( \phi_{X_i} \), the intrinsic spin function \( \mathcal{X}_i \) and the isospin function \( \tau_i \). In this version of AMD the isospin function is fixed to be up for protons and down for neutrons. Thus, an AMD wave function is parametrised by a set of complex parameters \( Z \equiv \{ X_{ni}, \xi_i \}(n = 1, 3 \text{ and } i = 1, A) \), where the \( X_i \)'s indicate the centres of Gaussians for the spatial part and the \( \xi_i \)'s are the parameters for the directions of the intrinsic spins.
Figure 3.1: Excitation energies for the levels of $^{10}$Be [KE98b]. The theoretical results of the variational calculations after spin and parity projection (VAP) in the AMD (right) are compared with the experimental data (left) from Ref. [Rag89]. Density distributions of protons (neutrons) for the intrinsic states are also displayed in the left (right) columns of the black panels.

In the simplest version of AMD for the study of nuclear structure, the ground-state wave function of a system is obtained by the energy variation (using a cooling procedure to obtain the lowest state) of the parity projected eigenstate from a Slater determinant. Furthermore, the direction of intrinsic spins of single-particle wave functions are fixed to be up or down, $\xi_i = \pm \frac{1}{2}$, for simplicity. Therefore, the spin-isospin functions of the single-particle wave functions are chosen as $p^\uparrow$, $p^\downarrow$, $n^\uparrow$ and $n^\downarrow$ in the initial state and are fixed during the energy variation. In this case the total wave function of a system is parametrised only
by $X \equiv \{X_1, X_2, ..., X_A\}$, which are the centroids of Gaussian wave packets in the phase space. Finally, the state has to be projected to obtain states of good parity ($\pi$) and spin.

$$\Phi^\pi(X) = (1 \pm P)\Phi_{AMD}(X)$$

The minimum-energy state obtained with the energy variation for the parity projected state is considered as the intrinsic state of the system. After the intrinsic wave function is projected to obtain the total angular momentum, the expectation values of the electromagnetic operators and intrinsic density distributions of the nucleons can be obtained.

An example of these calculations is shown in Fig. 3.1. In the figure the excitation energy of the levels in $^{10}$Be are shown. The density distribution of protons is shown in the left part of the coloured panels and for the neutrons in the right part. As can be seen from the figure the protons and the neutrons are grouped into two parts which makes two $\alpha$-particles. Also, the neutron distribution for the levels is different with likely $\sigma$ and $\pi$ bond structure (see Fig. 3.2).

Higher excited states are constructed by superimposing two bases in order to orthogonalise to the lower states. In this sense, in “the simplest version of AMD”, as for the variational calculations after the parity projection, the full information on the excitation energy spectra is obtained.

### 3.1.3 Electromagnetic transition probabilities

When a nucleus emits $\gamma$ rays they are produced by the electromagnetic radiation field which can be described in terms of a multipole expansion [Bla59]. Here, only some basic aspects of multipole transitions which are used in this work are discussed. If the nucleus decays from a state with spin $I_i$ to a state with spin $I_f$, the total transition probability is given by:

$$T_{fi} = \frac{8\pi(L + 1)}{\hbar L((2L + 1)!!)^2} \left(\frac{E_\gamma}{\hbar c}\right)^{2L+1} B(\lambda L, I_i \rightarrow I_f)$$

(3.2)
Figure 3.2: In the upper part of the picture, the density distributions of the valence neutron in the $0^+_1$ (top left) and $0^+_2$ (top right) states in $^{10}\text{Be}$ [KE98b] are shown. The bottom part shows the corresponding schematic figures of the molecular orbits surrounding two clusters in the $\pi$ and $\sigma$ bonds respectively.

where the reduced transition probabilities $B(\lambda L)$ are given by

$$B(EL, I_i \rightarrow I_f) = \frac{1}{2I_i + 1} |\langle f \| \hat{Q} \| i \rangle|^2$$

for the electric case and

$$B(ML, I_i \rightarrow I_f) = \frac{1}{2I_i + 1} |\langle f \| \hat{M} \| i \rangle|^2$$

for the magnetic case. Here $\hat{Q}$ and $\hat{M}$ are the electric and magnetic multipole operators, respectively. It is assumed that the size of the radiation source (the nuclear radius) is much smaller that the wavelength of the emitted photon (the so called “long-wave approximation”). In order to take into account the different possible orientations of the angular momentum, $L$, an average over the initial and the final $m$-state values is taken. A detailed derivation of these equations can be found in [Rin80] and [Mor76].

The question of how many nucleons contribute to the radiation is very much related to the nuclear structure of the relevant states. There are two extreme
cases. The single-particle aspect, with the assumption that only one nucleon is excited and the collective aspect in which a large number of nucleons contribute coherently to the radiation.

Weisskopf estimates

Beginning with the single-particle picture of the shell model where only one proton moves from one state to another, the transition probabilities \( T(\lambda L) \) of multipolarity \( L \) can be calculated in the case of electric transitions as:

\[
T(EL) = \frac{8\pi(L + 1)}{\hbar L((2L + 1)!!)^2} \frac{e^2}{4\pi\epsilon_0\hbar c} \left( \frac{E_\gamma}{\hbar c} \right)^{2L+1} \left( \frac{3}{L + 3} \right)^2 cR^{2L} \tag{3.5}
\]

and for magnetic transitions:

\[
T(ML) = \frac{8\pi(L + 1)}{\hbar L((2L + 1)!!)^2} \left( \mu_p - \frac{1}{L + 1} \right)^2 \left( \frac{\hbar}{m_p c} \right)^2 \left( \frac{se^2}{4\pi\epsilon_0\hbar c} \right) \left( \frac{E_\gamma}{\hbar c} \right)^{2L+1} \left( \frac{3}{L + 3} \right)^2 cR^{2L-2} \tag{3.6}
\]

where \( \mu_p \) is the magnetic moment of the proton, \( m_p \) is the proton mass and the wave functions of the states are obtained using a square well potential. By setting the \( \left( \mu_p - \frac{1}{L + 1} \right)^2 \) term to be equal to 10 and by taking \( R = R_0 A^{1/3} \), estimates can be made for the lower multipole orders. These are known as the Weisskopf single-particle estimates, and are given in Table 3.1

<table>
<thead>
<tr>
<th>Weisskopf estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T^W(E1) = 1.02 \times 10^{14} A^{2/3}E^3 )</td>
</tr>
<tr>
<td>( T^W(E2) = 7.23 \times 10^7 A^{4/3}E^5 )</td>
</tr>
<tr>
<td>( T^W(E3) = 3.37 \times 10^1 A^2E^7 )</td>
</tr>
<tr>
<td>( T^W(E4) = 1.06 \times 10^{-5} A^{8/3}E^9 )</td>
</tr>
</tbody>
</table>

Table 3.1: The Weisskopf estimates for the transition probabilities \( T(\lambda L) \) in units of \( s^{-1} \), where \( \lambda \) is \( E \) or \( M \) and the energy values, \( E \), are in MeV.
It is obvious that these estimates do not give a realistic calculation for the experimental transition rates, but they can be used as a scale. A transition is very often characterised by its transition probability in units of the Weisskopf estimate,

\[ F_W = \frac{T(\lambda L)}{T_W(\lambda L)} = \frac{B(\lambda L)}{B_W(\lambda L)} \]  

(3.7)

which is called the “favoured factor” or the “enhancement factor”. Clearly, the Weisskopf estimate is very useful when an order of magnitude indication is of interest.

**Collective transition probabilities**

The collective electric and magnetic transition rates for rotational nuclei were deduced by Bohr and Mottelson [Boh75]. Of interest for this work are mainly the \( E1 \) and \( E2 \) transitions. These are given by

\[ B(E1, I_i \rightarrow I_f) = \frac{3}{4\pi} e^2 D_0^2 \langle I_i K_i 10 | I_f K_f \rangle \]  

(3.8)

and

\[ B(E2, I_i \rightarrow I_f) = \frac{5}{16\pi} e^2 Q_0^2 \langle I_i K_i 20 | I_f K_f \rangle \]  

(3.9)

where \( D_0 \) and \( Q_0 \) are the intrinsic dipole and electric quadrupole moments, respectively and \( K \) is the projection of the total angular momentum on the symmetry axis. The expressions in angled brackets are the Clebsch-Gordan coefficients.

In Table 3.2 the collective transition rate estimates given in terms of the reduced transition probabilities for the lowest multiplicities are shown. The estimates are obtained by multiplying out the constant terms in Equation 3.2. They are used in this work to obtain the transition probabilities for the observed and previously measured \( \gamma \)-ray transitions and for those for which only theoretically predicted \( B(\lambda L) \) values are available. The results are given in Table 3.3.
Collective transition probabilities

<table>
<thead>
<tr>
<th>Transition</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T(E1)$</td>
<td>$1.59 \times 10^{15} E^3 B(E1)$</td>
</tr>
<tr>
<td>$T(E2)$</td>
<td>$1.22 \times 10^9 E^5 B(E2)$</td>
</tr>
<tr>
<td>$T(E3)$</td>
<td>$5.67 \times 10^2 E^7 B(E3)$</td>
</tr>
<tr>
<td>$T(E4)$</td>
<td>$1.69 \times 10^{-4} E^9 B(E4)$</td>
</tr>
<tr>
<td>$T(M1)$</td>
<td>$1.76 \times 10^{13} E^3 B(M1)$</td>
</tr>
<tr>
<td>$T(M2)$</td>
<td>$1.35 \times 10^7 E^5 B(M2)$</td>
</tr>
<tr>
<td>$T(M3)$</td>
<td>$6.28 \times 10^0 E^7 B(M3)$</td>
</tr>
<tr>
<td>$T(M4)$</td>
<td>$1.87 \times 10^{-6} E^9 B(M4)$</td>
</tr>
</tbody>
</table>

Table 3.2: Relation between the transition probability $T(\lambda L)$ and the reduced transition probability $B(\lambda L)$. Here $T(\lambda L)$ are in units of $s^{-1}$, $B(EL)$'s in $e^2 fm^{2i}$ and $B(ML)$ in $\mu_N^2 fm^{2i-2}$.

The ratio between the electric dipole and the electric quadrupole moment can be extracted from the experimental data using the ratio of the reduced transition probabilities $B(E1)/B(E2)$. If the intensities of the $E1$ and $E2$ transitions are $i_{E1}$ and $i_{E2}$, respectively, then the $B(E1)/B(E2)$ ratio can be calculated using

$$\frac{i_{E1}}{i_{E2}} = \frac{1.59 \times 10^{15} E^3_{\gamma(E1)} B(E1)}{1.22 \times 10^9 E^5_{\gamma(E2)} B(E2)}.$$  \hspace{1cm} (3.10)

The ratio of the intrinsic dipole to the intrinsic quadrupole moment, $[D_0/Q_0]$, can be extracted using Equations (3.8) and (3.9) and the obtained $B(E1)/B(E2)$ ratio has the following form

$$\frac{B(E1)}{B(E2)} = 2.4 \left[ \frac{D_0}{Q_0} \frac{\langle I_i K_{i10} | I_f K_f \rangle}{\langle I_i K_{i20} | I_f K_f \rangle} \right]^{2}.$$  \hspace{1cm} (3.11)

### 3.2 Search for $\gamma$-ray decays

For the isotopes of beryllium ($^{10-12}$Be) detailed $\gamma$-ray spectroscopic studies have not been reported before. With the advent of the new highly efficient $\gamma$-ray detector arrays a new search has been started. The nuclei are deformed in their ground states [vO97, KE95b] and it is expected that with the adding of further ‘valence’ neutrons to these deformed states a variety of new strongly deformed
Table 3.3: Branching ratios ($\Gamma_\gamma/\Gamma_{tot}$) for $\gamma$-ray decays in $^{10,11,12}$Be. The values are calculated using the formulae for the transition probabilities (see Section 3.1.3) and $\Gamma = \hbar/\tau$, where $\Gamma_\gamma$ and $\Gamma_{tot}$ are the $\gamma$-ray and total widths of the levels respectively and $\tau$ is the mean lifetime. Here, $J\pi_i$ is the spin and parity of the initial state and $J\pi_f$ is the spin and parity of the final state for a transition with multipolarity, $\lambda L$. Energies marked with ‘*’ correspond to theoretical $B(\lambda L)$ values. A detailed discussion of the transitions is given in Section 3.2.
states can be formed. Recent discussions of the structure of some light neutron-rich nuclei have focused attention on the fact that strong clustering occurs close to the single-particle and cluster emission thresholds. The most likely structure of the strongly deformed shapes is related to the $\alpha$ clustering. Possible weak $\gamma$-ray decay branches are also expected in states above the particle thresholds. Measuring the $\gamma$-ray branches from these states will give a direct measure of their deformation.

Of particular interest is the search for $\gamma$-ray decay branches related to the dimer structure created by two $\alpha$ particles. Possible new $\gamma$-ray transitions as well as the rotational bands in the beryllium isotopes are shown in Fig. 3.3 and discussed below.

The electromagnetic transition probabilities between low-lying states and the ground state in $^9$Be have been measured only by electron inelastic scattering. Decay widths indicate that some of the corresponding $\gamma$-ray branches are within reach of the new Ge-detector-array facilities; for example the $5/2^- \rightarrow 3/2^- (g.s.)$ transition has $\Gamma_\gamma = (8.9 \pm 1.0) \times 10^{-2}$eV which gives a branching ratio of $\Gamma_\gamma/\Gamma_{tot} \sim 10^{-4}$.

### 3.2.1 Expected transitions in $^{10}$Be

Even though the $\gamma$-spectroscopy of $^{10}$Be was made 30 years ago [Rou69], there is still great interest in weak $\gamma$-decay branches from states above the particle threshold, that could be measured. Some of them are given below.

$E_\gamma=1.108$ MeV (M1), $3/2^- (7.37$ MeV$) \rightarrow 2/2^- (6.236$ MeV$)$

This transition should have a large $B(M1)$ value, since both the initial and final states belong to the same rotational band, though none of the theoretical models of $^{10}$Be can predict the exact number. The $2^- \rightarrow 1^-$ transition between the lowest members of the same band was studied decades ago [Rou69], but the result is not
Figure 3.3: Plot of the excitation energies of known states in $^{9-12}$Be grouped into rotational bands where observed $\gamma$-ray transitions are shown with blue lines. This compilation is made in order to show the relation of the isomeric (molecular) shapes, expected close to the particle threshold. Possible new $\gamma$ decays are indicated by red lines.

The $0^+_2$ state in $^{10}$Be is long-lived ($\tau_m=1.1$ ps) and it is assumed to be the band-head of a molecular dimer band. It lives unusually long compared to the other lower lying states (with a typical lifetime of the order of femtoseconds), because of its shape, and $\gamma$-ray decays from the levels of this band with energies above the particle threshold will be very fast and compete with a retarded particle decay. The $2^+_3$ state is very narrow ($\Gamma=6$ keV) situated 700 keV above the threshold for neutron emission. It is assumed to have a deformed structure and there are some

$E_\gamma=1.363$ MeV (E2), $2^+_3(7.542$ MeV)$\rightarrow 0^+_2(6.179$ MeV)
hints that it decays through the $\alpha + ^6\text{He}$ channel. The neutron decay of this state is retarded due to the particular shape of the band relative to the ground state of $^9\text{Be}$. Itagaki and Okabe [Ita00] predict a huge $B(E2)$ value of $35.72 \, e^2 \, fm^4$ for this transition, in contrast to the transition to the ground state which is predicted to have $B(E2)=0.19 \, e^2 \, fm^4$. AMD calculations [KE99] also predict a large $B(E2)$ value for this transition.

$$E_\gamma=1.413 \text{ MeV (E2)}; \, 3^-_1 (7.371 \text{ MeV}) \rightarrow 1^-_1 (5.958 \text{ MeV})$$

Kanada-En’yo et al. [KE95b] predict a rather large transition rate for this $\gamma$-ray branch, $B(E2)=11.2 \, e^2 \, fm^4$, which reflects the fact that both the initial and final states are members of the same deformed rotational band.

### 3.2.2 Expected transitions in $^{11}\text{Be}$

A similar situation as that discussed for $^{10}\text{Be}$ is expected. Possible $\gamma$-ray transitions are shown in Fig. 3.3 and discussed below:

$$E_\gamma=1.29 \text{ MeV (E2)}; \, 5/2^-_2 (5.25 \text{ MeV}) \rightarrow 3/2^-_3 (3.96 \text{ MeV})$$

These states are the first two states of the $K=3/2$ band which should be well deformed so that very large E2 transition rates (as for $^{10}\text{Be}$) can be expected.

$$E_\gamma=3.64 \text{ MeV (E2)}; \, 3/2^-_3 (3.96 \text{ MeV}) \rightarrow 1/2^-_1 (0.32 \text{ MeV})$$

The total width of the 3.96 MeV state is so small that it is not observable in charged particle spectroscopy [Boh99]. Since the AMD-calculations [KE95b] predict that the $B(E2)$ value for this transition should be relatively large (8.13 $e^2 \, fm^4$), it is likely that the branching ratio for $\gamma$-ray emission could be large enough to detect.
\[ E_\gamma = 4.93 \text{ MeV (E2)}, \quad 5/2^- (5.25 \text{ MeV}) \rightarrow 1/2^- (0.32 \text{ MeV}) \]

The first excited state of the K=3/2 band at 3.96 MeV should have a larger particle-decay width than the second excited state (at 5.25 MeV), but the competing \( \gamma \)-decay may still be detectable, as the predicted \( B(E2) \) value (8.06 \( e^2 f m^4 \) [KE95b]) is relatively large.

### 3.2.3 Expected transitions in \(^{12}\text{Be}\)

Similar molecular states to those discussed for \(^{10}\text{Be}\) and \(^{11}\text{Be}\) with large deformations are predicted [KE95b] for \(^{12}\text{Be}\). These states should have a \( \gamma \)-ray branch to the ground state or to the first excited \( 2^+ \) state.

\[ E_\gamma = 2.10 \text{ MeV (E2)}, \quad 2^+ (2.10 \text{ MeV}) \rightarrow 0^+_1 \text{(g.s.)} \]

Kanada En’yo et al. [KE95b] predict \( B(E2) = 14 \ e^2 f m^4 \) for this transition. They also calculate a large number of new molecular states predicted at low excitation energies. These states should have a large probability to decay through \( \gamma \)-emission. Descouvemont and Baye [Des01] calculate a transition rate of just 6.6 Wu using microscopic \( \alpha + ^8\text{He} \) and \(^6\text{He} + ^6\text{He} \) wave functions for these levels, but they also predict two rotational bands based on \( 0^+_2 \) and \( 1^-_1 \) states with large probabilities for intraband transitions. For example, the \( 3^-_1 \rightarrow 1^-_1 \) transition has a predicted value of \( B(E2) = 31.1 \) Wu.

\[ E_\gamma = 0.90 \text{ MeV (E2)}, \quad 2^+_3 (7.3 \text{ MeV}) \rightarrow 0^+_2 (6.4 \text{ MeV}) \]

For the second K\( ^\pi = 0^+_1 \) band, which consists of the closed \( p \)-shell states, Kanada En’yo et al. [KE95b] predict a small \( B(E2) \) value of \( 7 \ e^2 f m^4 \), while Descouvemont and Baye [Des01] predict 23.4 \( e^2 f m^4 \).

The experiment described below was made to search for the weak \( \gamma \)-decay properties of these beryllium nuclei.
3.3 The $^7\text{Li} + ^{10}\text{Be}$ experiment

The population of strongly deformed structures in direct reactions is possible if the projectile and/or target already exhibit strong clustering. Accordingly, a neutron-rich $^{10}\text{Be}$ target and a beam of $^7\text{Li}$ have been chosen. A thick target experiment has been performed at the Laboratori Nazionali di Legnaro (LNL) in Italy, bombarding a $^{10}\text{BeO}$ target with a $^7\text{Li}$ beam. Gamma-ray events were collected using GASP in conjunction with the ISIS charged particle detector (for details see Chapter 2). This enabled channel selection by the observation of $\gamma-\gamma$-particle coincidences. For example the following reactions can be observed:

$$(^7\text{Li}+^{10}\text{Be}) \rightarrow (^4\text{He}+\text{p})+^{12}\text{Be}$$

$$(^4\text{He}+\text{np})+^{11}\text{Be}$$

$$(^4\text{He}+2\text{np})+^{10}\text{Be}$$

$$(^4\text{He})+^{13}\text{B}$$

$$(^3\text{He}; \text{d}+\text{p})+^{14}\text{B}$$

$$(2\text{p})+^{15}\text{B}$$

$$(\text{p})+^{16}\text{C}$$

The total thickness of the Be-Oxide target [Goo74b], which has an enrichment of $>94\%$ in $^{10}\text{Be}$, is 600 $\mu\text{g/cm}^2$ with 230 $\mu\text{g/cm}^2$ of $^{10}\text{Be}$. The target is deposited on a 4.9 mg/cm$^2$ of platinum. In order to stop the beryllium and boron recoils a thick (40 mg/cm$^2$) Au backing was added. To suppress the reactions on the Au backing a beam energy of 29.4 MeV was chosen to be below the Coulomb barrier for the $^7\text{Li} + ^{197}\text{Au}$ system. To protect the silicon detectors against elastically scattered $^7\text{Li}$-particles aluminium-absorber foils, were used.

In this experiment, the reconstruction of the kinematics (see Section 2.3.3.) using the emitted $\alpha$ particles is not used because the recoiling nuclei stop in the target. Moreover the break-up reactions of $^7\text{Li}$ on the backing resulted in a huge amount of chance light particle events in the ISIS detectors which leads to reduced statistics in the $\gamma-\gamma$-particle coincidences. One should keep in mind that the $^{10}\text{Be}$ target was in the form of BeO and this results in a large amount
A simple Doppler shift correction for the recoiling nuclei in the beam direction by varying $\beta$ and optimising the peak width for certain nuclei was made. The advantage of such a thick target experiment is that the full range of emitted recoils is accepted and that for $\gamma$-ray transitions emitted from excited states which have lifetimes longer than the stopping time ($\sim$1 ps), no Doppler shift correction is needed.

### 3.4 Results and discussion

The results presented below are from the experiment described in Section 3.3, using the $^{10}$BeO target and the $^7$Li beam. The reactions on $^{16}$O yielded new results about the band structure in the mirror nuclei $^{21}$Ne and $^{21}$Na (see Chapter 4), but contaminated the data for the beryllium isotopes. Despite this and the break up reactions on the backing of the $^{10}$Be target $\gamma$-ray transitions from the bandheads of the proposed molecular rotational bands in $^{10}$Be were observed.

#### 3.4.1 $^{10}$Be

**Intensities of $\gamma$-ray decays**

Levels both under and above the particle emission threshold in $^{10}$Be were populated and the $\gamma$-decay scheme for the levels under the particle threshold was extracted (see Fig. 3.4). A representative spectrum gated by the 3367 keV transition from the $2^+$ to the ground state is shown in Fig. 3.5.

The $2^-$ level at an excitation energy of 6.26 MeV is several keV under the $^9$Be+$n$ threshold. Since the decays of all other levels under the threshold are observed except this one it is likely that the negative parity states are not populated in this reaction or that the $2^-$ level has a very short lifetime (less than femtoseconds) making the expected strong $E1$ (99% $\gamma$-ray branch) transition to
Figure 3.4: Decay-scheme for $^{10}$Be as observed in this experiment. The widths of the arrows correspond to the relative intensities of the $\gamma$-ray transitions. Energies are in keV.
CHAPTER 3. SPECTROSCOPY OF BERYLLIUM ISOTOPES

Figure 3.5: The top panel shows the γ-ray spectrum gated by the 3367 keV transition to the ground state in $^{10}$Be. The lines at the beginning of the spectrum ($\leq 1000$ keV) are due to reactions on the backing of the target (Au and Pt) and $^{16}$O. The bottom panel shows the expanded part of the spectrum, clearly showing the 2590 keV doublet.
the $2^+_1$ state at an excitation energy of 3367 MeV very broad in the observed spectra. Since the other observed negative parity state ($1^-_1$) at 5.959 MeV forms a doublet with the $2^+_2$ state, it is impossible to say if the $1^-_1$ level is populated via $\gamma$-ray decay from other negative parity states or directly in the reaction. However, the $1^-_1$ state discussed here is populated from the isomeric $0^+_2$ state at 6.18 MeV. This leads to a sharp component in the observed $\gamma$-ray coincidence spectra, see, for example, Fig. 3.6.

![Gate on 219 keV](image)

Figure 3.6: Gamma-ray spectrum gated by the 219 keV $E1$ transition from the isomeric $0^+_2$ state at an excitation energy of 6.18 MeV in $^{10}$Be. The sharp components of the subsequent transitions can be clearly seen.

To deduce the relative intensities of the $\gamma$-ray transitions in $^{10}$Be, an arbitrary, normalised intensity of 100 was taken for the 3367 keV $2^+_1 \rightarrow 0^+$ transition (see Table 3.4).

The intensities of the transitions were (when possible) directly deduced from the gate on the 3367 keV ground-state band transition, assuming $I(2811) + I(2590$
Table 3.4: Relative intensities of the $^{10}\text{Be}$ transitions as deduced from the $^7\text{Li}+^{10}\text{Be}$ reaction, normalised to the 3367 keV, $2^+ \rightarrow 0^+$ transition. The doublet transitions are marked with ‘*’ and the possible new 219 keV doublet transition with (a).

From these data the branching ratio of the $1^-$ decay is different from the published value. Instead of a ratio of 83:17 [AS88], the 2591 and 5959 keV transitions are approximately equally strong. Unfortunately, it is difficult to give an exact ratio because of the large background, but the difference between these two transitions is not greater than 10 percent. The energy difference between the $1^-$ and the $2^+_2$ levels is $1.5 \pm 0.5$ keV. This means that the newly observed 219 keV transition could be a doublet decaying to both the 5959 and 5957 keV levels. When gating on the 219 keV transition (see Fig. 3.6), the 2591 keV decay is more intense than expected from the literature [AS88].

In addition, the 3367 keV transition appears broadened when gating on the 2591 keV transitions (see Fig. 3.7). This suggests that the $2^+_2$ and/or $1^-_1$ levels are populated directly in the reaction or via prompt $\gamma$-ray decays from higher lying states. It is possible that this population is the origin of the difference in
the measured branching ratios.

Since the $0^+_2$ isomeric state at an excitation energy of 6178 keV is very weakly populated via neutron decay from $^{11}\text{Be}$ (see Fig. 3.8), in the current data it is likely to be populated from higher lying levels, above the particle threshold. Such transitions have not been observed, perhaps because the initial levels are very broad giving rise to a large variation in the subsequent $\gamma$-ray decays.

The probability of populating the $2^+_2$ level through inelastic scattering can be negated since the 5957 keV transition from the $2^+_2$ level directly to the ground state is very weak. In addition, the possible population from the $0^+_2$ state is also very weak.

In the current work, it was proven that the $2^+_2$ bandhead (at 5957 keV) was populated by observing the linking $\gamma$-ray transitions to the ground-state band. This also confirmed the population of the proposed molecular bands in $^{10}\text{Be}$ (see Fig. 3.3) since the states at 5957, 5959 and 6178 keV are all considered as bandheads of molecular structures. After $\beta$-decay of $^{11}\text{Li}$ to $^{11}\text{Be}$ the direct population of the $2^+_2$ and $1^-_2$ levels, via neutron decay has not been observed [Mor97]. The neutron decay of $^{11}\text{Be}$ mainly populates the ground state and the first excited state in $^{10}\text{Be}$ (see Fig. 3.8).

Below, various aspects relating to the cluster structure of particular states in $^{10}\text{Be}$ will be discussed.

**The ground state ($0^+$) and the $2^+$ (3.37 MeV)**

Both of these states are strongly populated in various one-nucleon transfer reactions on $^9\text{Be}$, suggesting that their structure is not very different. The strong E2 transition ($B(E2)=10.5\pm1.2\ e^2fm^4$) between the $2^+$ and the $0^+$ states is an indication of their large deformation. Furthermore, new calculations from Kanada En’yo et al. [KE99] and Itagaki et al. [Ita00] show that both states are likely to have a cluster structure. The two $\alpha$-clusters here are closer together than in
Figure 3.7: Top panel: $\gamma$-ray spectrum gated by the 2811 keV transition from the isomeric $0^+_2$ state. The sharp nature of the 3367 keV transition is clear. Bottom panel: $\gamma$-ray spectrum gated by the 2591 keV (doublet) transition in $^{10}$Be. The 3367 keV transition now appears broadened indicating a short ($\sim$femtoseconds) lifetime component, perhaps as a result of direct population in the reaction or prompt $\gamma$-ray feeding.
Figure 3.8: A schematic diagram of the neutron and γ-ray transitions taken from [Mor97]. On the left, the β-decay branching ratios from $^{11}\text{Li}$ to energy levels in $^{11}\text{Be}$. On the far right are the energy levels in the neutron-decay daughter $^{10}\text{Be}$. The energies of the states are given in MeV and their widths are qualitatively indicated by the line thickness.
Be which is expected since the energy difference between these two levels, 3.37 MeV in $^{10}$Be, is bigger than the corresponding value in $^8$Be (3.04 MeV). In the current work a strong E2 transition (3367 keV) between these two states, the $2^+$ at 3367 keV and the ground state in $^{10}$Be, was observed, which is in good agreement with previous experimental results.

$2^+_2$ (5.958 MeV), $1^- (5.959$ MeV), $0^+_2 (6.179$ MeV) and $2^- (6.263$ MeV)

These states are important since they prove the existence of a two-centre molecular structure in this A=10 nucleus [vO96b, vO97]. Earlier, the $0^+_2$ state in $^{10}$Be was an object of special interest, as it was considered as a candidate for an intruder level, having a configuration of two neutrons in the sd-shell. This was confirmed by the results from one-nucleon transfer reactions to which its contributions were either weak, or unobservable. In recent cluster and molecular approximations it was suggested that the 6.18 MeV state ought to be an extended object with a well developed $\alpha$-2n-$\alpha$ cluster structure [vO96a, vO97, KE99, Ita00, Oga00]. In Ref. [KE99] it was found that this state should also have a developed $\alpha$-2n-$\alpha$ structure, but according to Ref. [Oga00], a small $(0.01)$ $\alpha$-$^6$He spectroscopic factor is obtained.

$3^- (7.37$ MeV) and $2^+ (7.54$ MeV)

These two states are considered to be the first and second excited members of two different rotational bands, based on the $1^-_1$ and $0^+_2$ configurations, respectively. Both were found as resonances in the total neutron capture cross-section of $^9$Be, there, the $3^-$ state has a strong population and a width of $\Gamma=15.7$ keV [AS88]. The $2^+$ state is weakly populated and has $\Gamma=6.3$ keV [Boc51, Fos61, Lan64]. In the current work $\gamma$-ray transitions de-populating these levels were not observed. The reason for this is likely to be that the calculated $\Gamma_\gamma/\Gamma_{tot}$ ratio (see Table 3.3) is not reachable with the experimental set up used here. Alternatively, it could be that the transitions coming from these states are very broadened.
3.4.2 $^{11}\text{Be}$

Internal $\gamma$-ray decays from $^{11}\text{Be}$ were not observed. The neutron emission threshold in this nucleus is rather low (500 keV) and up to now there is only one known level which decays to the ground state emitting a 320 keV ($1/2^- \rightarrow 1/2^+$) $\gamma$-ray. As can be seen from Fig. 3.9, in the current data the peak corresponding to this transition was not visible in the total projection spectrum. Since there are no other known transitions in this nucleus it was not possible to continue the investigation.

![Figure 3.9: The part of the total $\gamma$-ray projection spectrum showing where the known transition at 320 keV to the ground state in $^{11}\text{Be}$ should be.](image)

3.4.3 $^{12}\text{Be}$

This nucleus has been studied recently [Shi03] and the experimental results show previously unobserved $\gamma$-ray transitions and prove the existence of an isomeric $0^+_2$ state at an excitation energy of 2.24 MeV, which decays to the first excited
2\(^{+}\) state at 2.10 MeV following the emission of a 140 keV \(\gamma\) ray. This 140 keV transition was observed in coincidence with the known 2.10 MeV \((2^{+}_{1} \rightarrow 0^{+}_{1})\) transition. The isomeric state was produced by projectile fragmentation of \(^{18}\)O on a beryllium target [Shi03]. Looking again at other experimental data from Fortune \textit{et al.} who measured the \(^{10}\)Be(t,p)\(^{12}\)Be reaction [For94], the population of the isomeric state is very weak. Consequently this state will not be populated (or only very weakly populated) in transfer reactions.

In the present data this energy region is difficult to reach because of contaminant background lines. Even if peaks are seen at the same energy (140 keV and 2100 keV) it is not possible to extract more information. One such contaminant is one of the \(\gamma\)-ray transitions to the ground state in \(^{18}\)F, which is at the same energy (2101 keV).

### 3.4.4 Gamma-ray coincidence data for the carbon isotopes

Gamma-ray transitions from known levels in \(^{13}\)C and \(^{14}\)C have also been observed (see Figs. 3.10 and 3.11). All transitions up to the particle thresholds in both nuclei were observed.

### 3.4.5 Summary and outlook

The \(\gamma\)-ray decay properties of neutron-rich isotopes of beryllium, close to the particle emission thresholds, have been studied. The results obtained are in agreement with the theoretical calculations. Some of the levels, even those expected to have large \(\gamma\)-ray transition probabilities are still beyond the reach of modern germanium detector arrays. For example, the 1.363 MeV transition in \(^{10}\)Be has a \(B(E2)\) value of 35.72 \(e^2 fm^4\), but since this level is above the particle emission threshold the \(\Gamma_{\gamma}/\Gamma_{tot}\) ratio is \(2.1 \times 10^{-8}\) (see Table 3.3). Nevertheless, for the first time a complete \(\gamma\)-ray decay scheme for the levels up to the neutron
decay threshold has been constructed. By comparing existing data to the current results, a new $\gamma$-ray transition from the isomeric $0^+_2$ state in $^{10}$Be was found. Furthermore, the population of the bandheads of the proposed rotational bands from levels above the particle thresholds was proven. Since these levels, the $2^+_2$ at 5958 keV and the $0^+_2$ at 6179 keV, are strongly populated and the $\gamma$-ray transitions above the particle thresholds feeding them are very weak, a direct population via neutron decay of $^{12}$Be and $^{11}$Be must be assumed from the 2n stripping reaction $^{10}$Be($^7$Li, $\alpha$p)$^{12}$Be$^*$. The results of this experiment have been the basis for another experiment with a similar motivation, which was carried out recently and the data are still
Figure 3.11: Decay-scheme for $^{14}$C as observed in this experiment. Energies are in keV.
under analysis. The new experimental set up includes the EUROBALL german-
nium array and the BRS (Berlin Binary Reaction Spectrometer) [Thu98] and is likely to detect some of the γ-ray transitions above the particle emission threshold described in Chapter 3.2. Such an experimental set-up enables γ-γ-particle and particle-particle-γ coincidences to be measured. The selection of binary reactions enables a powerful pre-selection of the reaction channels to be made, implying that the EUROBALL array can be used mostly with one fold γ-ray multiplicities or with γ-γ-ray events, for which the total efficiency is 10 times higher than for GASP. The position information from the two BRS-detectors and the high granularity of Euroball can be used to achieve a high quality Doppler shift correction.
Chapter 4

Spectroscopy of neon isotopes

4.1 Theoretical consideration

In this chapter the formation of asymmetric molecular structures in neon isotopes is discussed. The structure of some bands in $^{21}$Ne can be interpreted as consisting of an intrinsic asymmetric structure ($^{16}$O+$\alpha$) bound by a covalent neutron in $\sigma$ and $\pi$ orbitals. The observed parity doublet states in $^{21}$Ne in the present experiment are discussed and a corresponding band structure for the states up to 12 MeV excitation energy in $^{21}$Ne is given.

4.1.1 Cluster model

In the context of the covalently bound molecular structures in nuclei, the cluster structure of $^{21}$Ne will be based on the underlying structure of $^{20}$Ne. The cluster structure in $^{20}$Ne has been discussed extensively in Refs. [But96, Ohk98, Kim01, Duf94]. In Ref. [Ohk98] it is shown that a shallow local potential, which is phase equivalent to the deep potential obtained in a double folding model [Abe93], gives an appropriate description of the scattering of $\alpha$-particles on $^{16}$O as well as explaining some some of the deformed rotational bands in $^{20}$Ne [Ohk98, Abe93, Buc75]. The strong repulsion obtained in the potential at
small distances can be interpreted (similar to the case of the $\alpha+\alpha$ system) as being due to the Pauli exclusion principle. Calculations based on the antisymmetrised molecular dynamics approach for $^{20}\text{Ne}$ performed by Horiuchi [Hor72, Kim01] also show the pronounced clustering. In the latter reference it is shown how the $^{16}\text{O}+\alpha$ clustering related to the octupole degree of freedom, develops with increasing quadrupole deformation; the clustering appears for values of $\beta_2 \approx 0.32$.

4.1.2 Reflection asymmetric shapes

This approach was used to describe the structure of nuclei for the first time in the 1950's [Aza53, Ste54, Ste55] following the observation of low-lying negative parity states in actinide nuclei.

The origin of the octupole deformation can be understood from the single-particle level energy sequence for a harmonic-oscillator potential. In certain cases an orbit is lowered into the next lowest major shell by the $l^2$ and $l.s$ terms. These \textit{intruder orbits} $(l_{\text{int}},j_{\text{int}})$, lie close to orbits with $l = l_{\text{int}} - 3$ and $j = j_{\text{int}} - 3$, and the pairs of orbits with $\Delta l = \Delta j = 3$ can be strongly coupled by the octupole interaction.

Mean-field theories and algebraic models [But96] and macroscopic-microscopic methods [Lea75] suggest that reflection asymmetric shapes are observed in the lightest nuclei in the sd-shell and for even-even $N = Z$ nuclei, respectively. Experimentally this prediction can be supported by the observation of negative parity states in the middle of the sd-shell, where the shell model does not predict a large number of negative parity states [vO01, Des93]. These are typically intrinsic octupole shapes. The same observation of parity doublets is predicted for dinuclear systems (molecules) consisting of two clusters with unequal masses [Her50]. Such shapes, related to the observation of bands of opposite parity connected by strong $E1$ transitions, are well known in heavy odd mass nuclei.
Simplex quantum number

A reflection-symmetric nucleus has Parity, $\pi$, which describes the symmetry under space reflection, and Signature, $\sigma$, which describes the invariance with respect to a rotation of $180^\circ$ about the rotation axis, which are good quantum numbers. For a reflection-asymmetric intrinsic structure the nucleus breaks the parity and signature symmetries, which means that both signature and parity are no longer good quantum numbers. The remaining symmetry is a combination of both and known as simplex [Goo74a, Naz85], which is still a good quantum number. The simplex is equivalent to a reflection in a symmetry plane (the plane containing the symmetry axis), and it is defined as the eigenvalues of the $S$ operator

$$S = PR^{-1}. \quad (4.1)$$

In terms of simplex a rotational band having a simplex, $s$, has states of spin $I$ with alternating parity, related by [Boh75],

$$p = se^{-i\pi I}. \quad (4.2)$$

For a reflection asymmetric nucleus with even mass, the spin and parity sequences are restricted to

$$s = +1, \quad I^\pi = 0^+, 1^-, 2^+, 3^-, ..., \quad (4.3)$$

and for the case of an odd mass nucleus

$$s = +i, \quad I^\pi = \frac{1^+}{2}, \frac{3^-}{2}, \frac{5^+}{2}, \frac{7^-}{2}, ..., \quad (4.4)$$

Parity splitting

For the first time alternating-parity rotational bands with interleaved states in the sequence $I^+, (I+1)^-, (I+2)^+, ..., $ were observed in the actinide region in $^{218}$Ra and $^{222}$Th [FN82, War83, Bon83], which were related to octupole deformation.
Figure 4.1 shows three different plots of potential energy, $V$, versus the octupole deformation, $\beta_3$. The top potential corresponds to a rigid spheroidal nucleus, which is axially symmetric as well as reflection symmetric in its ground state, but can undergo fluctuations (vibrations) about this symmetric shape. If the vibration is of odd multipolarity, then the nucleus can take on a reflection-asymmetric shape giving a $K^\pi = 1^-\gamma$ band at an energy of approximately 1MeV.

The potential at the bottom of Fig. 4.1 represents a nucleus with a permanent ground state octupole deformation ("a rigid pear shaped nucleus"). The potential barrier has in this case two degenerate minima at $\pm \beta_3$ and rises to infinity at $\beta_3=0$. The nucleus can not take on a reflection-symmetric shape and the energy level spectra of such a nucleus with even mass is characterised by a set of perfectly interleaved state of alternating parity ("parity inversion doublets"). The case of the potential in the centre of Fig. 4.1 is intermediate between the two other potentials. There is a small potential barrier (see Fig. 4.1) and the nucleus can tunnel through the barrier to the mirror image shape. This interaction results in the displacement of the two bands. In reality, the limit shown by the bottom potential is not reachable, and the barrier separating the two degenerate minima is more like that pictured in the centre. The displacement of a state from the middle point of the two neighbouring states with opposite parity is known as the parity splitting, $\delta E$, and can be calculated by

$$\delta E = E(I)^- - \frac{1}{2}(E(I+1)^+ + E(I-1)^+).$$

In some nuclei it is observed that the parity splitting tends to zero at spins around $10\hbar$, giving energy level schemes similar to those shown in the lower part of Fig. 4.1. It seems that the rotation acts to stabilise the octupole deformation. Two possible explanations of this phenomena are: (i) the octupole shape has weaker pairing correlations, which increases the moment of inertia and (ii) the rotational motion perturbs the single-particle states of opposite parity, which brings the $\Delta l = \Delta j=3$ orbitals closer together, with increasing ro-
tional frequency, thereby enhancing the strength of the octupole correlations [Naz87, Egi90, Naz92].

Figure 4.1: Plots of the potential energy and the associated energy level spectra for different axially symmetric ($K=0$) shapes with octupole deformation. The top potential represents a nucleus with reflection-symmetric ground state shape. The centre potential is an intermediate case between this form and the static octupole-deformed shape, which corresponds to the bottom potential.

In odd nuclei reflection asymmetric shapes are manifested through the appearance of doublets of states with opposite parities with an energy splitting depending on the height of the internal barrier separating the two reflected configurations. Such intrinsic parity-violating shapes, related to bands of opposite parity, for example for $^{19}$F and $^{21}$Ne, have also been interpreted as possible evidence for cluster and molecular structures [vO01, Des93]. The energy surface can have two minima in the octupole degree of freedom. This is schematically
illustrated in Fig. 4.2, where two shapes with $\beta_3 < 0$ and $\beta_3 > 0$ are separated by a potential barrier. A rearrangement of nucleons, or a tunneling process by the nucleons, will bring the system from the state with $\beta_3 > 0$ to the other shape, the reflected configuration with $\beta_3 < 0$.

A particularly interesting case which has recently been discussed in the context of the clustering or octupole scenario, is the nucleus $^{21}\text{Ne}$ [vO01]. As for the case of $^{20}\text{Ne}$, the inverted doublet structure which, based on the $\alpha + ^{16}\text{O}$, has been discussed in 1972 by Horiuchi et al. [Hor72].

In $^{21}\text{Ne}$ this parity doublet structure is mirrored in two parity doublets of bands with $K = 3/2$ and $K = 1/2$. Although mixing of the corresponding positive parity states with states of quadrupole deformation is to be expected, the negative parity states are difficult to relate to simple Nilsson orbitals, suggesting an interpretation based on cluster configurations. In most papers on the structure of light sd-shell nuclei the negative parity states have been omitted in the discussion of the level structure e.g. Refs. [Hof89, Jia92], because they invoke complicated particle excitations to the p-shell as well as to the f-shell.

![Figure 4.2](image)

Figure 4.2: The potential $V(\beta_3, I)$ for fixed angular momentum, $I$, as a function of $\beta_3$ showing the two symmetric minima in the proposed bands in $^{21}\text{Na}$. For the $K = 3/2$ bands the lower barrier induces a large energy splitting; for $K = 1/2$ the higher internal barrier gives degenerate positive and negative parity bands.
The ground state structure of $^{21}$Ne has been described in cluster calculations using the microscopic three-cluster generator coordinate method [Des93].

One of the descriptions is based on the mixing of different $^{16}$O+$\alpha$+n configurations, which give rise to an intrinsic reflection asymmetric shape, whereas in others a reflection symmetric shape of the $\alpha+^{12}$C+$\alpha$+n structure is preferred.

Having in mind the $\gamma$-decay properties connected to reflection asymmetric (octupole) shapes, we have investigated the rotational structure of $^{21}$Ne and $^{21}$Na. The result is that the excited states of $^{21}$Ne are consistent with the formation of a molecular structure with a stable dipole moment. An estimate of the intrinsic dipole moment based on $\gamma$-ray branches gives a limiting value in agreement with an intrinsic $^{16}$O+$^4$He+n configuration.

**Octupole bands in $^{20}$Ne and $^{21}$Ne**

The structure of $^{20}$Ne has also been discussed within the framework of the deformed Nilsson model suggesting stable quadrupole and octupole deformations [But96, Buc75]. Similarly, rotational bands as parity doublets using cluster models and octupole shapes have been obtained in other light nuclei such as $^{18}$O [Rei93] and $^{19}$F [Kra69, Duf94].

Experimentally, two opposite parity bands are observed in $^{20}$Ne: the negative parity band starting with a $1^-$ state at 5.787 MeV, and the positive parity band being the ground-state band. This observed structure supports the identification of the two bands as ‘inversion doublets’ of an intrinsic reflection asymmetric octupole with $K = 0$ and parities $\pi = (+$ and $-)$. The energy splitting between the two bands is approximately 5 MeV. Further states in higher lying bands in $^{20}$Ne of positive and negative parity are obtained using the $\alpha + ^{16}$O model by Ohkubo et al. [Ohk98].

For $^{21}$Ne this concept of octupole deformation and a weak coupling of the cluster shape with an extra valence neutron leads to a close analogy to spin parity doublets with $K = 3/2$ and $K = 1/2$. Such ‘parity doublets’, in the context of a
reflection asymmetric molecular structure and one covalent molecular orbital for neutrons, are discussed in Ref. [vO01].

4.2 Experiments: $^{18}\text{O}+^{13}\text{C}$ and $^7\text{Li}+^{16}\text{O}$ reactions

In the present experiment excited states of $^{21}\text{Ne}$ and $^{21}\text{Na}$ have been populated in the $^7\text{Li}+^{16}\text{O}$ reaction from the $^7\text{Li}+^{10}\text{BeO}$ experiment, described in Section 3.3.

Since the performed experiment was optimised to investigate $\gamma$-ray transitions in isotopes of Be, B and C, and the energy was chosen to be below the Coulomb barrier on the nuclei in the backing to avoid reactions, only low angular momenta states were populated. In Fig. 4.3 are shown all $\gamma$-ray transitions in $^{21}\text{Ne}$ from states populated in the $^7\text{Li}+^{16}\text{O}$ reaction.

![Figure 4.3: Decay scheme of $^{21}\text{Ne}$ showing the levels not forming rotational structures together with the $K^\pi=3/2^+$ ground-state band as observed in the present work. All energies are given in keV. The intensities (widths) of the arrows are not to scale, except for the $K^\pi=3/2^+$ band.](image)

Meanwhile, another experiment discussed in this work was performed with the reaction $^{13}\text{C}(^{18}\text{O},2\alpha2n)^{21}\text{Ne}$ at a beam energy of 100 MeV in order to enhance the population of higher spin levels in particular the $K = 1/2$ band members.
Figure 4.4: Gamma-ray decay scheme for $^{21}$Ne as observed in the $^{18}$O+$^{13}$C reaction. All energies are given in keV.
In this purely compound nucleus reaction only yrast and nearly yrast states are strongly populated (see Fig. 4.4).

The results obtained for the yrast band decay from this experiment, which are shown in Fig. 4.4 are in good agreement with the results from the first $^7\text{Li}+^{16}\text{O}$ experiment.

### 4.3 Results and discussion

#### 4.3.1 Octupole bands in $^{21}\text{Ne}$

To underline the proposed rotational character of bands in $^{21}\text{Ne}$, excited states from previous work and the newly observed states in this work have been arranged into rotational-like structures with $K$ values of $1/2$ and $3/2$. In Fig. 4.5 are shown the pairs of opposite parity states of the proposed octupole configurations arranged in parity doublets (see details in Ref. [vO01]). For the $K=1/2$ bands the higher lying states are slightly different from Ref. [vO01].

In Fig. 4.5, additional levels associated with structures corresponding to the 'normal' quadrupole-deformation are not included. These have been discussed in previous studies, which have established the main features of the $\gamma$-decay in this nucleus [Hof89, Rol71, Rol72, Kuh74, War71, And81, Pil72].

The most striking characteristic for the $K=1/2$ bands, besides the small energy splitting of the parity doublets, is the long lifetime (110 ps) of the $1/2^-$ state at 2.79 MeV decaying into the $K=3/2$ ground state band.

In the cluster description this requires a re-arrangement from a $\pi$-type neutron orbital (see Section 3.1.2.) with densities outside the symmetry axis to a $\sigma$-type bond [vO01], where neutrons are concentrated along the symmetry axis.

With a lifetime of 110 ps the competing M2 and E1 transitions connecting the $1/2^-$ state to the $5/2^+$ and the $3/2^+$ states of the $K=3/2$ ground-state band respectively, thus appear to be retarded by more than three orders of mag-
Figure 4.5: Rotational band structure of $^{21}$Ne proposed here, showing the parity doublet structure (based on Fig. 4 of Ref. [vO01]). All energies are given in keV. The parity of the states follow the band assignments.

The intensity ratio from the presented experimental data for the two transitions with 2789 keV (E1) and 2438 keV (M2) is $E1/M2 = 3$. Warburton et al. [War71] have measured this ratio and obtained a value of 5.7. Using the common systematics of the relative strength of E1 and M2 transitions, the E1 transition would be expected to be stronger than the M2 transition by a factor of $10^6$. Whereas in the data presented here it is stronger only by a factor of 3. The strong suppression of the E1 transition once more indicates that the configurations are not related to single centre (mean-field) structures, but to two centre molecular configurations. Two possible explanations were already discussed in Ref. [War71]. The explanation of this fact, which is discussed in this work, is that...
the enhanced de-excitation through an M2 transition is related to the change of
two quantum numbers going from the $1/2^-$ state ($K^\pi = 1/2^-$ band) to the $5/2^+$
or to the $3/2^+$ state ($K^\pi = 3/2^+$ band), namely the change of the orbital angular
momentum and the relative spin orientation, in order to change the $K$-value. The
almost degenerate $I^\pi = 1/2^+$ and $1/2^-$ levels and consequently the small energy
splitting in the $K = 1/2$ bands suggests that the internal energy barrier between
$\beta_3 < 0$ and $\beta_3 > 0$ for such a change of configuration is quite high (see Fig. 4.2).

In Fig. 4.8 the observed transitions between states of the proposed octupole
character are shown in the newly ordered band structure. Fig. 4.6 shows the
remaining transitions observed in the $^7\text{Li} + ^{16}\text{O}$ experiment, together with the
ground-state band, namely the part of the level scheme of $^{21}\text{Ne}$ which does not
seem to be connected to the band structures discussed above.

The $K^\pi = 3/2^+$ ground-state band has been observed with intraband transi-
tions up to the $I^\pi = 13/2^+$ state at an excitation energy of 6448 keV. The states
in the $K^\pi = 3/2^-$ band have been identified through their interband decays to the
$K^\pi = 3/2^+$ band. No intraband $\gamma$-transitions have been observed in the negative
parity band. The lowest state in this band ($3/2^-$, 3664 keV) also feeds the $1/2^-$
state of the $K^\pi = 1/2^-$ band, with an intensity, a factor of three lower when
compared to its transition to the $K^\pi = 3/2^+$ band. The fact that the members
of the $K^\pi = 3/2^-$ band decay predominantly into states of the $K^\pi = 3/2^+$ band
is consistent with the fact that we are dealing with strongly enhanced E1 tran-
sitions. The existence of an E1 transition connecting the $I^\pi=3/2^-$ bandhead to
the ground state could not be clearly established due to the absence of a feeding
transition on which to gate and due to the poor selectivity of the ungated spectra.

The transitions de-populating the states differing by intervals of 2 units of spin
from the $K = 3/2$ bandhead, namely $J^\pi = 3/2^-$, $7/2^-$, $11/2^-$, $15/2^-$, are clearly
visible (see Fig. 4.8), whereas the transitions de-populating the interleaved $5/2^-$,
$9/2^-$ states (and the unobserved $13/2^-$) are much weaker. The $13/2^- \rightarrow 11/2^+$
transition could not be observed in the present data. This is probably due to
Figure 4.6: Gamma-ray decays observed in the $^{16}\text{O}({}^7\text{Li},\text{np})$ reaction showing the proposed $K = 3/2$ and $K = 1/2$ bands of $^{21}\text{Ne}$. All energies are given in keV. The intensities (widths) of arrows to the right side of the ground-state band are not to scale.
the rather low γ-ray branch in this region of excitation energy ($E_x > 9$ MeV), as it is inferred from very low $\Gamma_\gamma/\Gamma$-values measured by Billowes et al. [Bil87]. (The only exception is the $15/2^-$ level at 11.988 MeV excitation energy.) The $5/2^- \rightarrow 5/2^+$, 3533 keV transition could not be confirmed because of its partial overlap with the 3545 keV γ-ray transition de-exciting the state at 6412 keV (see Fig. 4.8).

The non-observation of the intraband $E2$ transitions allows an estimate to be made of the ratio of the observed $B(E1)/B(E2)$ branching ratios. The intrinsic dipole moment is related to the $E1$ transition probability by the rotational model formula (see section 3.1.3)

$$B(E1, I_i \rightarrow I_f) = \frac{3}{4\pi} e^2 D_0^2 |\langle I_i K_i 10 | I_f K_f \rangle|^2,$$

and the $E2$ transition probability to the quadrupole moment by

$$B(E2, I_i \rightarrow I_f) = \frac{5}{16\pi} e^2 Q_0^2 |\langle I_i K_i 20 | I_f K_f \rangle|^2.$$

These formulae provide a consistent method to extract the intrinsic dipole moments from the experimental data, although the assumption that the investigated nucleus is a good rotor may be questionable. The experimentally measured intrinsic dipole moments in several regions of the nuclear chart can be found in the review article of Butler and Nazarewicz [But96].

Assuming for $^{21}$Ne, that the same value of the measured intrinsic quadrupole moment of $^{20}$Ne occurs, namely, $Q_0 = 58(3) \ e.f.m^2$ [Hor71], the intrinsic dipole moment for $^{21}$Ne is expected to be $D_0 > 0.1 \ e.f.m$. This large value for $D_0$ is indicative of stable octupole deformation, as expected for a reflection asymmetric $^{16}\text{O}+\alpha+n$ structure.

The $K^\pi = 1/2^+$ band was visible in the data only by its weak transitions from the bandhead to the lowest states ($3/2^+$ and $5/2^+$) in the $K^\pi = 3/2^+$ band. No feeding of the $K = 1/2^+$ bandhead by γ-ray decays from other members of this band could be identified, although its decay is observed.
The de-population of the 6032 keV $9/2^-$ state, which is proposed to be a member of the $K = 1/2^-$ band, into the $9/2^+$ and $7/2^+$ levels of the $K = 3/2^+$ band has also been observed. Some mixing between the state of the $K = 3/2^-$ band at 6639 keV excitation energy and the $K = 1/2^-$ band member at 6039 keV, both with spin $9/2^-$, cannot be excluded.

For the $K^{\pi} = 1/2^-$ bandhead, fed by the $J^{\pi} = 3/2^- (K = 3/2^-)$ state, transitions de-populating the $1/2^-$ level were observed. Further feeding of the bandhead was not observed. From this observation and the absence of $\gamma$-ray feeding of the $K = 3/2^-$ band and the $K = 1/2^+$ bandhead, a conclusion could be made, that the population patterns observed in this experiment, give very strong indications for the direct population of these states. This direct population must originate from an $\alpha$-transfer followed by the transfer of a neutron, leaving two nucleons (a neutron and proton) to be emitted in a direct process.

The population of the low-spin states of the $K = 1/2$ bands is remarkable and indicates the formation of cluster states, because the ($^7Li, np$) or ($^7Li, d$) reactions favour states with higher spin, due to the angular momentum mismatch between incoming and outgoing channels. A similarly strong population of cluster states via $^5He$ has been observed in $^{14}C$ in a recent study of the $^9Be(^7Li, d)^{14}C$ reaction using a high resolution Q3D-spectrometer [Boh02].

### 4.3.2 Angular distributions and DCO ratios

A nucleus formed in a fusion-evaporation reaction is in a state with its angular momentum vector perpendicular to the axis of the beam direction. Even after the subsequent evaporation of particles the residual nucleus keeps a high degree of orientation for a long time (of the order of nanoseconds). If $\gamma$ rays are emitted from a nucleus in such a state, the angular distribution of the relative intensities (with respect to the beam axis) depends on the multipolarity of the transitions. This distribution [Sie65] is given by the equation:
\[ I(\theta) = \sum_{\ell=\text{even}} A_{\ell} P_{\ell}(\cos \theta) \]  

(4.8)

where \( P_{\ell}(\cos \theta) \) are the Legendre polynomials and \( A_{\ell} \) are their coefficients. The coefficients are tabulated (see, for example, Ref. [dM74]).

Figure 4.7 shows the angular distributions for pure quadrupole and dipole transitions. By examining the relative \( \gamma \)-ray coincidence intensities at angles approximating 0\(^\circ\) (for example) and 90\(^\circ\) it is possible for different multipoles to be distinguished. This method is called Directional Correlations de-exciting Oriented states (DCO) [Kra73].

![Figure 4.7: Intensity distributions for a dipole (\( \Delta I=1 \)) transition (solid line) and a quadrupole (\( \Delta I=2 \)) transition (dashed line) as a function of the angle \( \theta \), with respect to the beam direction. (Zero degrees corresponds to the positive x-axis.)](image)

The spins and parities of the \(^{21}\)Ne levels were deduced, where possible, using the DCO analysis. Special attention was paid to the DCO ratio for charged particle coincidence events. For the data obtained with the GASP detector array (see Chapter 2) a \( \gamma-\gamma \) matrix was created with \( \gamma \) rays from the germanium detectors at
90° with respect to the beam direction on one axis and the $\gamma$ rays from those detectors at 34° and 146° on the second axis. In order to reduce contamination from other reaction channels the data in the matrix have been sorted under the condition of being in coincidence with 0 and 1 proton detected in the ISIS Si-telescopes. Gates were set on both axes on known stretched quadrupole E2 transitions and the intensities of the transitions of interest were extracted from the resulting spectra. The theoretical DCO ratios, $R_{DCO} = I_{\gamma}(90°)/I_{\gamma}(34°)$, in the GASP geometry are $R_{DCO} \approx 1.0$ for stretched quadrupole (E2), and $R_{DCO} \approx 2.0$ for pure dipole (E1 or M1) transitions. However, if the gates are set on a pure dipole transition, the expected DCO ratios for quadrupole and for dipole transitions are $\approx 0.5$ and $\approx 1.0$, respectively. Table 4.1 shows the results of the DCO analysis for the transitions, where the analysis was possible. Most of the published spin assignments could be confirmed. The cases where a new spin assignment is possible, are marked with (*).

In Table 4.2 the new spin assignments obtained from the experimental data are listed. The assignment of the 7040 keV and 7422 keV need to be confirmed. In the recent Nuclear Data Tables [ht02], a state at 6552 keV, de-populating via a 4806 keV $\gamma$ ray to the 1746 keV, $7/2^+$ state, is published. It is possible that this is the same state as at 6543 keV with here the same transition as the 4797 keV $\gamma$-ray.

4.3.3 The mirror nuclei $^{21}\text{Ne}$ and $^{21}\text{Na}$

The background subtracted spectra relevant for the $\gamma$-ray decays observed in $^{21}\text{Na}$ and $^{21}\text{Ne}$ are shown in Figs. 4.8 and 4.9, both gated on the lowest $5/2^+ \rightarrow 3/2^+$ (g.s.) transition in the respective nuclei. The Doppler-shift correction has been done by an optimisation procedure of the recoil velocity since the lifetimes of the states, except for the long-lived $5/2^+$ and $1/2^-$ levels at 351 keV and 2789 keV respectively, in $^{21}\text{Ne}$, and the $5/2^+$ level at 332 keV in $^{21}\text{Na}$, were shorter than
Table 4.1: DCO ratios for transitions in $^{21}$Ne, gated by the $5/2^+ \rightarrow 3/2^+$ transition at 351 keV.

<table>
<thead>
<tr>
<th>$E_\gamma$ [keV]</th>
<th>$J_\pi^i \rightarrow J_\pi^f$</th>
<th>$R_{DCO}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1121</td>
<td>$9/2^+ \rightarrow 7/2^+$</td>
<td>1.00 ± 0.01</td>
</tr>
<tr>
<td>1395</td>
<td>$7/2^+ \rightarrow 5/2^+$</td>
<td>1.10 ± 0.01</td>
</tr>
<tr>
<td>1566</td>
<td>$11/2^+ \rightarrow 9/2^+$</td>
<td>1.06 ± 0.02</td>
</tr>
<tr>
<td>2687</td>
<td>$11/2^+ \rightarrow 7/2^+$</td>
<td>0.50 ± 0.02</td>
</tr>
<tr>
<td>3165</td>
<td>$(9/2^-) \rightarrow 9/2^+$</td>
<td>0.67 ± 0.05</td>
</tr>
<tr>
<td>4173*</td>
<td>$9/2^+ \rightarrow 9/2^+$</td>
<td>0.74 ± 0.06</td>
</tr>
<tr>
<td></td>
<td>$5/2^+ \rightarrow 5/2^+$</td>
<td></td>
</tr>
<tr>
<td>4286</td>
<td>$9/2^- \rightarrow 7/2^+$</td>
<td>1.05 ± 0.04</td>
</tr>
<tr>
<td>4555*</td>
<td>$9/2, 11/2 \rightarrow 9/2^+$</td>
<td>0.69 ± 0.02</td>
</tr>
<tr>
<td>4797*</td>
<td>$9/2 \rightarrow 7/2^+$</td>
<td>1.29 ± 0.11</td>
</tr>
<tr>
<td>4985</td>
<td>$7/2^- \rightarrow 5/2^+$</td>
<td>0.86 ± 0.08</td>
</tr>
<tr>
<td>5093*</td>
<td>$11/2^{(-)} \rightarrow 9/2^+$</td>
<td>1.04 ± 0.08</td>
</tr>
</tbody>
</table>

Table 4.2: Newly derived spin assignments obtained from the recent experiment as deduced from the DCO ratios (marked with (*) in Table 4.1). The third column shows the assignments before this work.

<table>
<thead>
<tr>
<th>$E$ [keV]</th>
<th>$J^i$ (this work)</th>
<th>$J^\pi$ (former assignment [htt02])</th>
</tr>
</thead>
<tbody>
<tr>
<td>6543</td>
<td>9/2</td>
<td>(see text)</td>
</tr>
<tr>
<td>7040</td>
<td>9/2$^+$</td>
<td>(5/2, 9/2$^+$)</td>
</tr>
<tr>
<td>7422</td>
<td>9/2, 11/2</td>
<td>(7/2$^+$, 11/2$^-$)</td>
</tr>
<tr>
<td>7960</td>
<td>11/2$^{(-)}$</td>
<td>(new state)</td>
</tr>
</tbody>
</table>
the stopping time of the nuclei in the backing material.

![Gamma-ray spectrum](image)

Figure 4.8: Gamma-ray spectrum obtained by gating on the 351 keV \((5/2^+ \rightarrow 3/2^+)\) ground-state band transition in \(^{21}\)Ne. The labelled peaks are all placed in \(^{21}\)Ne.

For \(^{21}\)Na no peaks have been observed above 3 MeV. Comparing the peak intensities of \(^{21}\)Ne and \(^{21}\)Na in the energy region between 1 and 3 MeV and the peak strengths of \(^{21}\)Ne in the energy region above 3 MeV, peaks of \(^{21}\)Na (if existing) with a few tens of counts would be expected. Since there are no peaks observed above 3 MeV when gating on the lowest lying transition in \(^{21}\)Na, it is likely that there are no transitions from states populated in \(^{21}\)Na above 3 MeV in the present reaction in coincidence with the \(5/2^+ \rightarrow 3/2^+\) transition.

The nucleus \(^{21}\)Na is populated via the \((^7Li, 2n)\) reaction. Fig. 4.10 shows the level scheme and \(\gamma\)-ray decay pattern of \(^{21}\)Na as observed in the present experiment. Two new transitions have been added to the previously known level
Figure 4.9: Gamma-ray spectrum obtained by gating on the 332 keV \((5/2^+ \rightarrow 3/2^+)\) ground-state band transition in \(^{21}\text{Na}\). All labelled peaks belong to \(^{21}\text{Na}\) and no peaks are observed above 3 MeV.

scheme extending the ground-state \((K^\pi = 3/2^+)\) band up to a state at 4419 keV for which an \(I^\pi = 11/2^+\) assignment is proposed.

Since both \(^{21}\text{Ne}\) and \(^{21}\text{Na}\) are mirror nuclei, \(i.e.\) the neutron number of one equals the proton number of the other and vice versa, it can be seen by comparison of the two nuclei that the nuclear structure is very similar. This is a consequence of the fact that the nuclear force does not distinguish between neutrons and protons. Conversely, the existence of an additional proton, \(i.e.\) an additional charge, in one of the mirror nuclei \((^{21}\text{Na})\) causes a difference in the energy of the isobaric analogue states due to the Coulomb interaction.

Recently, the Coulomb energy difference (CED), the energy difference between isobaric analogue states defined as \(CED = E^*(^{21}\text{Na}) - E^*(^{21}\text{Ne})\), has been investigated in a series of mirror nuclei in the \(f_{7/2}\) shell, allowing the study of the spatial behaviour of the wave functions of the active valence nucleons \([\text{Ben98, Ekm00, Len01}]\). Since the Coulomb energy is only due to protons, it has been pointed out that when a \(J = 0\) proton pair (of the core) couples to another \(J\)-configuration, the Coulomb energy decreases \([\text{Ben98}]\). In the \(J = 0\) coupling the overlap of the two proton wave function (and therefore the Coulomb repulsion) is
Figure 4.10: Scheme showing the ground-state band and γ-ray transitions of $^{21}$Na as obtained in the present work. All energies are given in keV.
at a maximum. Moreover the detailed behaviour of the CED has been proven to be sensitive to the changes in the radii of the valence particles in different excited states [Len01].

Fig. 4.11 shows the CED for the $K^\pi = 3/2^+$ bands in $^{21}$Ne and $^{21}$Na from the present data. A decrease of the CED as function of spin up to $I=9/2\ h$, followed by an increase when approaching the maximum alignment of two particles in the $d_{5/2}$ orbit, was observed. The qualitative behaviour of the CED here is consistent with the result of antisymmetrised molecular dynamics calculations for $^{20}$Ne, where at higher spin the charge density becomes more compact.

![Figure 4.11: Coulomb Energy Difference (CED) between mirror states for the $K^\pi = 3/2^+$ band in $^{21}$Na and $^{21}$Ne (see text for details).](image)

The antisymmetrised molecular dynamics calculations by Kanada-En’yo et al. and Horiuchi et al. [KE95b, Hor95] predict a shrinking of the average radius of the core-cluster with increasing angular momentum. This shrinking of the radius in the $\alpha + ^{16}$O-system with spin, is also suggested by the result for the $\alpha + ^{16}$O potential of Ref. [Ohk98], where the potential minimum moves to smaller distances for larger $L$-values (see also Section 2.1 of Ref. [vO01]).
Taking into account the above mentioned mass distribution of the valence particles, it is likely that for the $K = 3/2$ configuration a covalent orbit of the $\pi$-type involves a proton or neutron distribution concentrated outside the symmetry axis. A shrinking of the core extension translates to an increased Coulomb repulsion for the covalently bound proton and corresponds to a decrease in the CED.

### 4.3.4 Gamma-ray decays and band structure in $^{22,23}\text{Ne}$

$^{22}\text{Ne}$

In this even-even nucleus, as can be seen from the Nilsson diagram (see Fig. 1.5) all nucleons are coupled and the spin of the ground state is $0^+$. In the $^{18}\text{O} + ^{13}\text{C}$ reaction studied here the ground-state band ($K^\pi = 0^+$) was strongly populated and 3 new $\gamma$-ray transitions and 2 new levels were found. The ground-state band was extended to higher spin ($8^+$). In Fig. 4.12 the $\gamma$-ray transitions in coincidence with the new transition from the $8^+$ level at 11030 keV excitation energy are shown. A level at 11032 keV excitation energy was measured in an $^{18}\text{O}(\alpha,\gamma)$ reaction. In Fig. 4.13 excitation energy versus $J(J+1)$ is plotted which shows that this nucleus is a very good rotor.

In this reaction as can be seen from Fig. 4.14 showing all $\gamma$-ray transitions gated by the decay to the ground state at 1274 keV, the rest of the populated states are likely to be intrinsic, based on single-particle excitations or rotational levels from other band structures.

Fitting the experimental data for the energy levels it is possible to obtain the rotational constant $\alpha$, which is connected with the moment of inertia $\Theta$ ($\alpha = \hbar/2\Theta$). The value obtained is 140 keV.
CHAPTER 4. SPECTROSCOPY OF NEON ISOTOPES

Figure 4.12: Gamma-decay spectrum gated by the new 4719 keV (8\(^+\) \rightarrow 6\(^+\)) transition in the \(K = 0^+\) ground-state band in \(^{22}\text{Ne}\).

Figure 4.13: Excitation energy versus \(J(J+1)\) plot for the positive parity yrast band in \(^{22}\text{Ne}\) as obtained from the current data.
Figure 4.14: Gamma-ray decay scheme for $^{22}$Ne as observed in this experiment. Energies are in keV.
\(^{23}\text{Ne}:\) deformation alignment

The mechanism of deformation alignment is capable of producing high-\(K\) rotational bands with either yrast status, as in the case presented in this work, or at marginally higher excitation energy in almost all deformed \(sd\) shell nuclei with an approximate balance between the number of protons and neutrons. The only exceptions are observed for \(A=19-22\).

The theoretical prediction [Roe00] (see Fig. 4.15) for the low-lying \(K^\pi=5/2^+\) rotational band is based on the deformation-aligned multi-particle configurations of the Nilsson-model. The band is based on the \((3/2)^2(5/2)^1\) configuration. The theoretically predicted members of the band lie at \(E_x= 0, 1702, 2517, 3931^*, 5926^*\) keV where the last two levels are from shell-model calculations. The members of the band as observed in the \(^{13}\text{C}(^{18}\text{O}, 2\alpha)^{23}\text{Ne}\) reaction are shown in Fig. 4.15 and 4.16. The level at 3843.3 keV excitation energy is already known, but there is no spin assignment and the next member of this band has not previously been observed.

As can be seen from Fig. 4.17, this nucleus is not a usual rigid rotor. Due to insufficient available experimental information it is not possible to obtain firm spin assignments in the current work. Taking into account the calculations made by Roepke [Roe00] and the obtained transition scheme it is likely that the observed states are members of the \(5/2\) band with spins \(5/2, 7/2, 9/2, 11/2\) and \(13/2\). According to Fig. 4.17 it is likely that there are two bands which cross at \(7/2\) \(\hbar\).

**Moments of inertia**

The moments of inertia extracted from the ground-state bands in \(^{21,22,23}\text{Ne}\) are almost the same (see Table 4.3), which suggests similar structures. According to this the neon isotopes can be described as \(^{20}\text{Ne}\)-core plus valence neutrons.
Figure 4.15: Gamma-ray decay scheme for $^{23}$Ne as observed in this experiment (middle). On the left side of the picture is shown the previously known information about this nucleus and on the right side the predicted band calculated in Ref. [Roe00]. Energies are in keV. The widths of the arrows are only proportional to the relative intensities for the ground-state band. The widths of the arrows are proportional to the intensities for the current data (middle).
Figure 4.16: Gamma-decay spectrum gated by the ground state 1702 keV $(7/2^+ \rightarrow 5/2^+)$ transition of the $K^\pi = 5/2^+, T = 3/2$ band in $^{23}$Ne.

Figure 4.17: Excitation energy versus $J(J+1)$ plot for the positive parity yrast band in $^{23}$Ne as obtained from the current data.
Table 4.3: Moments of inertia for the rotational bands in the neon isotopes.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>Bandhead</th>
<th>$\alpha$ [keV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{21}$Ne</td>
<td>$3/2^+, 3/2^-$</td>
<td>145, 141</td>
</tr>
<tr>
<td>$^{22}$Ne</td>
<td>0$^+$</td>
<td>149</td>
</tr>
<tr>
<td>$^{23}$Ne</td>
<td>5/2$^+$</td>
<td>140</td>
</tr>
</tbody>
</table>

4.4 Summary

The $\gamma$-ray decay of states populated in the $^{16}$O($^7$Li,2n) and $^{16}$O($^7$Li,np) reactions have been studied. The spectroscopy of $^{21}$Ne has been extended with respect to the concept of reflection asymmetric shapes due to octupole deformation. Two parity doublets with $K = 3/2, 1/2$ were established. The mirror nucleus to $^{21}$Ne, namely $^{21}$Na, was investigated and the behaviour of the Coulomb energy difference (CED) was interpreted.

Furthermore, the $\gamma$-ray decay properties of $^{21,22,23}$Ne, populated in the $^{18}$O($^{13}$C,xn) reaction, where $x=0,1,2$, have been studied. The decay scheme for the $K = 3/2$ band in $^{21}$Ne suggests a ‘pure’ compound nucleus reaction in which mainly yrast states are populated.

New $\gamma$-ray transitions in $^{22}$Ne and $^{23}$Ne have been identified and the level schemes have been extended to higher spins. Possible spin assignments for the new transitions have been suggested. The results show good agreement with theoretical shell-model calculations performed by Roepke [Roe00].
Chapter 5

Cluster emission

In nuclear structure studies the evaporation of $\alpha$-particles is a decay process which is commonly used to produce residual nuclei, which can be subsequently studied via $\gamma$-ray spectroscopy. Less frequently used is the emission of heavier charged clusters, a mechanism which is particularly interesting since it may be able to produce nuclei in states of angular momentum and excitation energy that are not normally populated via light-particle emission. Large $\gamma$-ray detector arrays like EUROBALL and GASP often make use of light charged-particle triggers to select a particular $\gamma$-decaying nucleus. The detection of $\gamma$-rays in coincidence with heavier clusters is generally more difficult requiring the integration of large gas counters with close-packed germanium-detector arrays. To overcome this difficulty we have studied the $\gamma$-ray decay of compound nuclei in coincidence with clusters, excited to states just above the particle threshold. As a trigger we have used the cluster decay into 2 or more $\alpha$-particles. The experiments have been performed at the Laboratori Nazionali di Legnaro, Italy, using the $\gamma$-detector array GASP, consisting of 40 high purity Ge detectors and a multiplicity filter of 80 BGO scintillators. Light charged particles have been detected by the ISIS silicon-ball consisting of 40 $\Delta E$-$E$ telescopes each covering a solid angle of about 0.20 sr (29° in the reaction plane). For more information see Chapter 2. Strongly correlated $\alpha$-particles emitted in the decay of such weakly bound cluster states
will be detected in the same Si telescopes and observed through the pile-up of signals.

Here, results from two different reactions will be presented with the aim of observing $^8\text{Be}$ and $^{12}\text{C}_{0^+_2}^*$ cluster emission.

1) $^{18}\text{O}+^{13}\text{C} \rightarrow ^{23}\text{Ne} + (^8\text{Be} \text{ or } 2\alpha)$, at an energy of $E_{\text{LAB}}(^{18}\text{O}) = 100$ MeV, aimed for the study of Ne isotopes. In the case of $^8\text{Be}$ emission one should take into account that we can not distinguish between the ground state, which is unbound by 91.9 keV and the first excited $2^+$ state. An estimation of the decay cone for the $^8\text{Be}$ (ground state) emission gives approximately $\approx 4^\circ$. Having a width of 1.5 MeV at an excitation energy of 3.04 MeV the excited $2^+$ level will produce an opening cone on average of $24^\circ$, this means that just a part of the $2\alpha$ pairs coming from the decay of such an excited state could be registered in one ISIS-telescope ($29^\circ$). Furthermore, the emission of $^6\text{Li}$ and $^7\text{Li}$ has been observed in this reaction.

2) $^{28}\text{Si}+^{24}\text{Mg} \rightarrow ^{40}\text{Ca} + (^{12}\text{C} \text{ or } 3\alpha)$ at an incident energy of 130 MeV. Due to the high incident energy the compound nuclear decay favours the emission of several $\alpha$-particles (up to 3 or 4 $\alpha$-particles) and the emission of $^8\text{Be}$ and $^{12}\text{C}$ fragments, as can be seen from the data shown in Fig. 5.1. This allows the observation of the reaction channel corresponding to the emission of $^8\text{Be}$ plus one $\alpha$-particle, as well as of the $^{12}\text{C}_{0^+_2}^*$-state which is localised at an excitation energy of 7.6542 MeV, 288 keV above the $3\alpha$ threshold. With the instantaneous break-up of this excited $0^+_2$-state, $3\alpha$ particles are emitted in a very narrow cone of only $\approx 10^\circ$, which fits well into the opening angle ($29^\circ$) of the ISIS-telescopes.
5.1 Experimental conditions

5.1.1 The ISIS detector and multiple hit events

Absorber foils of 12µm thick aluminium were mounted facing the target to prevent scattered beam particles from penetrating the silicon detectors. The observation of unbound clusters populated close to the decay threshold, such as in beryllium and carbon, becomes possible by the detection of their light decay products. The identification of the decay products, produced by the cluster decay, from the background of evaporated charged particles relies on the former being emitted in a narrow angular cone.
The total solid angle covered by the ISIS detectors is 64% of $4\pi$. Due to the width of the detector frames there is a gap of approximately 6-7° between adjacent detectors. The kinematic conditions (the narrow emission cone) allow us to neglect the contribution from the α-particle events arising from cluster decay products entering 2 neighbouring detectors.

Plotting the energy signal of the first (thin-ΔE) detector versus the signal of the second (thick-E) detector, the events, for each different $mZ^2$ value, following the Bethe-Bloch formula (see Eq. 2.2), will be separated into a distinct ‘banana’ shaped distribution (see Figs. 5.1 and 5.2).

Figure 5.2: A plot of ΔE-E signals from the ISIS telescopes obtained in the $^{18\text{O}}+$ $^{13\text{C}}$ experiment.

The majority of the 2α-events (97%) due to cluster decay are observed in
the first 3 rings of the ISIS detectors. Those events are registered as multiple
hit-signals in the identification plots, with 2- or 3-times higher values than the
original Bethe-Bloch-curves for the single $\alpha$-particles. In the emission of 2 or
3 $\alpha$-particles from an unbound state, each $\alpha$-particle obtains, in their centre of
mass frame, an energy of less than 100 keV, the original binary fragment being
emitted with an energy of typically 30-50 MeV. The momentum vectors of such
$\alpha$-particles are thus almost the same. As a consequence the multiple hit signals
corresponding to the detection of such strongly correlated decay products into the
same telescope lie on an event line which corresponds to a rescaling of the $\Delta E$-
and E-signals, i.e. having a functional dependence described by the Bethe-Bloch
formula.

For a quantitative evaluation the probability of having the so called ‘multiple
hit’ events must be discussed. The ISIS spectrometer was built to have a small
multiple hit event-rate of less than 2% even for high multiplicities Ref. [Far01].
In Fig. 5.3 the dependence of the probability for multiple hits on the efficiency
for different particle-multiplicities is shown. According to the data obtained
in the $^{18}\text{O}+^{13}\text{C}$ and $^{28}\text{Si}+^{24}\text{Mg}$ experiments the event-rate for 2 or 3 $\alpha$-particles
registered in the same detector compared to 2 or 3 $\alpha$-particles observed in different
detectors is around 30%.

For $N_D$ number of detectors, each covering a fraction $\varepsilon$ of the solid angle, the
probability of detecting $F$ (Fold) $\alpha$-particles of $M$ (Multiplicity) emitted will be:

$$P(F, M) = \binom{M}{F} (N_D\varepsilon)^F (1 - N_D\varepsilon)^{M-F}$$

(5.1)

In the case $M = F = 2$ this reduces to the total probability of detecting 2 out of
2 particles in all detectors:

$$P(2, 2) = (N_D\varepsilon)^2$$

(5.2)

The probability of detecting 2 out of 2 particles in separate detectors is:

$$P_{sh}(2, 2) = N_D(N_D - 1)\varepsilon^2$$

(5.3)
Figure 5.3: The probability of a multiple hit for a single segment covering a solid angle of Ω = 4π as a function of the efficiency of the segment, for various values of the emitted particle multiplicity (M) [Far01].

According to this the probability of double hits in single detectors will be:

\[
P_{dh}(2, 2) = P(2, 2) - P_{sh}(2, 2) = N_D \varepsilon^2
\]  

(5.4)

Thus, the ratio of chance double hits compared to registering the two particles in separate detectors (for equal and isotropic probability for emitting a single particle) is:

\[
\frac{P_{dh}(2, 2)}{P_{sh}(2, 2)} = \frac{1}{N_D - 1}
\]

(5.5)

which is independent of \( \varepsilon \).

One can assume that the probability of detecting 2α-particles coming from
\(^8\text{Be}\) is

\[
P(^[8\text{Be}]\text{)} = N_D\varepsilon \tag{5.6}
\]

Thus one can build the ratio of the probability of detecting 2 out of 2 particles in different detectors to the probability of detecting \(^8\text{Be}\):

\[
\frac{P_{sh}(2, 2)}{P(^[8\text{Be}]\text{)}} = (N_D - 1)\varepsilon \tag{5.7}
\]

The same considerations can be made for the emission of 3\(\alpha\) particles and \(^{12}\text{C}^*\). In this case for \(F=M=3\) the total probability is:

\[
P(3, 3) = (N_D\varepsilon)^3 \tag{5.8}
\]

For detecting three particles in different detectors in this case the probability is given by:

\[
P_{sh}(3, 3) = N_D(N_D - 1)(N_D - 2)\varepsilon^3 \tag{5.9}
\]

Thus, building the difference from these two equations (5.8 and 5.9) one can obtain the probability of triple hits:

\[
P_{trh}(3, 3) = P(3, 3) - P_{sh}(3, 3) = N_D(3N_D - 2)\varepsilon^3 \tag{5.10}
\]

For the detection of \(^{12}\text{C}^*\), since the particles are emitted together the probability is, as in the case of \(^8\text{Be}\),

\[
P(^[12\text{C}]\text{)} = N_D\varepsilon \tag{5.11}
\]

The contribution of random coincidences in the same telescope coming from uncorrelated evaporated \(\alpha\)-particles with different energies will be distributed over a wider region in the \(\Delta E\)-\(E\) plots. Here one observes a deviation from the Bethe-Bloch function as shown schematically in Fig. 5.4. The energy spectrum of the multiple hit events in this case is given by:

\[
N(E) = N_1(E_i)E_i + N_2(E_j)E_j \tag{5.12}
\]

For the same total energy, \(E = (E_i + E_j)\), the final distribution will not follow the Bethe-Bloch dependence of \(\Delta E \sim mZ^2/E\) because of the non linear dependence \((1/E)\) of the \(\Delta E\)-signal.
This deviation can be clearly seen by constructing the multiple hit events using 2 very different values of the energies for the 2 detected α-particles: in the case of an uncorrelated emission the energies of the two α-particles may be very different (B1 and B2 in Fig 5.4). Here, $E_{\text{multiple hit}} = B_1 + B_2$ (see Fig. 5.4). This implies that chance coincidences start to deviate towards higher $\Delta E$ values with respect to the rescaled Bethe-Bloch line of the $^8\text{Be}$. A similar effect is expected for the $^{12}\text{C}$-line, although here the probability of chance coincidences for 3 α-particles contributing to the cluster decay event line must be very small (see Fig. 5.1). In Figs. 5.1 and 5.2 the plots of the charged particle identification events are shown for the two reactions as was observed with the ISIS silicon-telescopes, which illustrate these considerations.

Figure 5.4: Schematic illustration of the deviation of the random multiple hit events from the event line of the coincident α-particles from the $^8\text{Be}$-emission. Two $\Delta E$-E-signals for different energies (cases B1 and B2) and for equal energies (case A), are chosen, as indicated.
In the case of $^8\text{Be}$ and $^{12}\text{C}$ emission the energies of the emitted $\alpha$-particles and the corresponding energy spectra are:

$$E_i = E_j = E_k$$

$$N_1(E_i) = N_2(E_j) = N_3(E_k) \quad (5.13)$$

For uncorrelated $\alpha$-particles evaporated with almost the same energies no deviation from the Bethe-Bloch formula will be observed. These events will lie in the $^8\text{Be}$' banana. According to Eq. 5.13, for the comparison between these multiple hit events and $^8\text{Be}$ or $^{12}\text{C}$ one can take the rescaled (by factor of 2 or 3 times respectively) energy spectrum for 1 $\alpha$-particle.

In addition to the true $^8\text{Be}$ and $^{12}\text{C}$ lines, there will still be events from sequential compound nucleus emissions which can produce chance coincidences, with events of 2 and 3 $\alpha$-particles with the same energies, which will not deviate from the rescaled Bethe-Bloch line to larger $\Delta E$-values. In order to assess these contributions a simple formula for chance coincidences, $N^{\text{random}}$, can be used.

$$N^{\text{random}}_{2\alpha}(E) = N^2_\alpha(E)\Delta\tau \quad (5.14)$$

The counting rates $N_\alpha$ can be taken from the running time and the total events in the $\alpha$-particle line. The integration time, $\Delta\tau$, in the electronic amplifier creating the signals is $\sim 90$ ns. Thus the calculated value for $N^{\text{random}}_{2\alpha}$ is less than 0.03% and can be neglected. This means that each event contains coincidences from just one reaction.
5.2 Results and discussion of cluster emission as a statistical process

5.2.1 Discussion of the charged particle spectra

Using the calibrated $\Delta E-E$-signals we have compared the total energy spectra of the reconstructed fragment emissions with the expected multiple random events due to the detection of 2 or more evaporated $\alpha$-particles. For the latter it is assumed that double or triple events are obtained by rescaling the energy of a single $\alpha$-particle by a factor of 2 or 3 respectively see Eq. 5.13. As shown in Figs. 5.5 and 5.6 the two curves have different shapes and maxima.

Thus, from Equation 5.5, looking at a channel with true multiplicity 2 it is not possible to extract the efficiency from the ratio, because this actually depends only on the number of detectors. Using only the 6 forward detectors (ring 1 in ISIS) the energy dependence of this ratio in the upper part of the spectrum for the emission of 2 $\alpha$-particles should tend towards 0.2 ($=1/(6-1)$). A deviation from this value will indicate that more processes contribute to the double hit, namely cluster emission.

The same sort of calculation for $M=3$ and $F=3$, gives for the ratio of triple hits and 3 $\alpha$-particles in different detectors 0.8 ($=18-2/((6-1)(6-2))$). Comparing the 2 or 3 $\alpha$-particles coming in different detectors with the 2 or 3 $\alpha$-particles coming from $^8\text{Be}$ or $^{12}\text{C}$ respectively, it is necessary to take care of the Jacobian (a matrix which transforms the ‘phase space’ of the centre of mass system into the ‘phase space’ of the laboratory system), which enters into the efficiency of the detectors for the laboratory system. Since the Jacobian is different for $^8\text{Be}$ ($^{12}\text{C}$) and for 2 (3) ‘independently emitted’ $\alpha$-particles it is hard to evaluate what the ratio should be at lower energies.

For the comparison of the observed energy spectra, taking into account the formulae for the transition probabilities written above, in the case of emission of
Figure 5.5: Upper panel: kinematical plots for the angular variation of the energy of 2 $\alpha$'s and of $^8$Be. Lower panel: normalised total energy spectra ($E_{\text{sum}} = \Delta E + E$) as observed with the ISIS-charged particle detector system for the emission of single $\alpha$'s, and of $^8$Be, in the reaction $^{18}$O+$^{13}$C.
Figure 5.6: Upper panel: kinematical plots for the angular variation of the energy of 3 α’s and of $^{12}\text{C}^*(0^+_2)$. Lower panel: total energy spectra ($\Delta E+E$-signals) as observed with the ISIS-charged particle detector system for the emission of 3 single α’s, and of $^{12}\text{C}^*(0^+_2)$ in the reaction $^{28}\text{Si}+^{24}\text{Mg}$.
Table 5.1: The different Jacobians for both reactions, ‘a’ for $^{18}\text{O} + ^{13}\text{C}$ and ‘b’ for $^{28}\text{Si}+^{24}\text{Mg}$.

<table>
<thead>
<tr>
<th>Jacobian</th>
<th>$J_{\alpha}^a$</th>
<th>$J_{\beta}^a$</th>
<th>$J_{\alpha}^b$</th>
<th>$J_{12C}^b$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.9</td>
<td>2.7</td>
<td>2.1</td>
<td>3.0</td>
</tr>
</tbody>
</table>

2 $\alpha$-particles one can obtain:

$$\frac{N_{8\text{Be}}^*}{N_{2\alpha}^*} = \frac{N_{8\text{Be}}}{N_{2\alpha}} \frac{1}{(N-1)} \frac{J_{8\text{Be}}}{\varepsilon(J_{\alpha}^\beta)^2}$$ (5.15)

where $N_{8\text{Be}}^*$ and $N_{2\alpha}^*$ are the experimentally observed counting rates for $^8\text{Be}$ and $2\alpha$ respectively and $J_{8\text{Be}}$ and $J_{\alpha}^\beta$ are the corresponding Jacobians. In Fig. 5.10 (lower panel) shows the experimentally deduced ratio between $^8\text{Be}$ and 2 $\alpha$-particles. It can be seen that the experimentally observed probability ratio between two statistical (uncorrelated) $\alpha$-particles in one detector and two $\alpha$-particles in two different detectors ($F=2$) is an order of magnitude smaller than the expected ratio (0.02 instead of 0.2). The explanation of this experimental fact is that in the emission of 2 $\alpha$-particles from a compound nucleus it is very unlikely that the second particle will be emitted in the same direction as the first one due to correlation effects; thus the main intensity observed is due to $^8\text{Be}$.

Another important aspect in the discussion of the energy spectra will be connected to the kinematics of the reactions with the emission of the total masses $A=4$, $A=8$ or $A=12$ fragments. The corresponding curves are shown in Fig. 5.5(upper panel) for the relevant angles of the first 3 ISIS rings of telescopes, containing $\approx97\%$ of the total multiple hit events. A similar calculation is shown for the second reaction in Fig. 5.6(upper panel). To obtain the $Q$-value it is necessary to assume a certain excitation energy of the residual nucleus. This implies a prior knowledge of the energy (on average), which is carried away. This can be taken from the systematics of compound decays studied in the same mass
region by Morgenstern et al. [Mor83]. In this work the average energy removed by an α-particle is deduced to be 22.1±1.5 MeV and for an individual nucleon 16.4±0.3 MeV. These values are in good agreement with most statistical model calculations.

After considering the different kinematical dependences of the binary emission process we can compare the spectra of $^8\text{Be}$ ($^{12}\text{C}$) with the rescaled single α-energy spectrum of the 2 (3)α-particles. This is shown in Fig. 5.5 for the ($^{18}\text{O}+^{13}\text{C}$) reaction and for the second reaction ($^{28}\text{Si}+^{24}\text{Mg}$), which shows the unbound $^{12}\text{C}$ fragment, in Fig. 5.6.

### 5.2.2 Gamma-ray coincidence spectra for different fragments

The γ-ray analysis for both reactions will enable the emission mechanism for the different channels to be discussed. In the off-line analysis the different reaction channels were selected by requiring that only the events corresponding to the detection of the proper number of α-particles and protons in the ΔE-E silicon telescopes were incremented into a symmetrised $E_{\gamma} - E_{\gamma}$ matrix.

**Gamma-ray coincidence spectra following the emission of $^8\text{Be}$ and $^{12}\text{C}$**

We are interested in the angular momentum and the energy balance of the binary processes. The first step is to look into the relevant γ-spectra which belong to a given residual nucleus and a given charged particle trigger. Generally, there are still many reaction channels within a charged-particle ISIS-trigger and additional γ-ray gates are needed to identify one residual nucleus uniquely. For the first reaction we have just 2 possible triggers - 2 α particles and $^8\text{Be}$ emission, but for the reaction $^{28}\text{Si}+^{24}\text{Mg} \rightarrow ^{40}\text{Ca}$ the γ-ray spectra can be obtained with 3 different charged particle triggers a) 3 α particles, b) $^8\text{Be}+\alpha$ particle emission, and c) $^{12}\text{C}^* (0^+_2)$ emission.
The relatively high velocity of the compound nucleus and the larger mass of the emitted fragments, means that the \(\gamma\)-ray spectra must be Doppler corrected. This correction was carried out by using the sum of the vectors of the coincidently registered charged particles (for more details see Section 2.3.3.). A satisfactory Doppler shift correction was obtained, resulting in an energy resolution of about 10 keV FWHM at 1000 keV.

First the \(^{28}\text{Si} + {^{24}\text{Mg}}\) reaction will be discussed, where \(^{8}\text{Be}\) and \(^{12}\text{C}^*_0\) emission was observed. The detailed spectroscopy of the dominant residual nuclei \(^{40}\text{Ca}\) and \(^{42}\text{Ca}\) is given in Ref. [Tor03]. Of particular interest here are the differences, which could be observed in the three various charged particle triggers leading to \(^{40-42}\text{Ca}\) as well as to \(^{39}\text{K}\). The results for the ungated \(\gamma\)-ray spectra with the different charged particle triggers are shown in Fig. 5.7.

In both reactions, as previously observed for \(^{8}\text{Be}\) emission [Thu99], the binary cluster channel carries less energy and less angular momentum from the residual nucleus than the sequential \(\alpha\)-particle emission. Therefore, subsequent neutron and/or proton evaporation becomes more conspicuous. This is particularly visible for the \(^{12}\text{C}\) emission (see Fig. 5.7), where the residual nucleus has enough energy even for the subsequent emission of 2\(p+2n\) or an \(\alpha\)-particle, which leads to \(^{36}\text{Ar}\).

The same result is observed in the \(^{18}\text{O} + {^{13}\text{C}}\) reaction (see Fig. 5.8), where the \(\gamma\)-ray spectrum gated on \(^{8}\text{Be}\) shows a stronger population for \(^{21}\text{Ne}\), which corresponds to a subsequent neutron evaporation, when compared to \(^{22}\text{Ne}\). Furthermore, using \(\gamma\)-particle coincidences it was possible to investigate the energy dependence of the relative population of these nuclei by gating on different energies for the 2 \(\alpha\)-particles, registered in the same detector. As can be seen from Fig. 5.9, the \(^{8}\text{Be}\) events are concentrated in the low energy region of the \(E_{\text{sum}}\). This is in good agreement with the curve shown in Fig. 5.10, corresponding to the ratio between the rates of the 2 \(\alpha\)-particles registered in different detectors and in the same detector, as function of the energy.

Having only statistical (uncorrelated) 2 or 3 \(\alpha\)-particles as multiple hit events...
Figure 5.7: Gamma-ray spectra obtained from the $^{28}\text{Si} + ^{24}\text{Mg}$ reaction gated by different charged particle triggers. Upper panel: Doppler corrected $\gamma$-ray spectrum gated by the $3\alpha$ channel. Lower panel: Doppler corrected $\gamma$-ray spectrum gated by the $^{12}\text{C}^*(0^+_2)$ channel.
Figure 5.8: Gamma spectra from the $^{18}$O+$^{13}$C reaction gated by different charged particle triggers. Upper panel: Doppler corrected $\gamma$-spectrum gated by the 2 $\alpha$ channel. Lower panel: Doppler corrected $\gamma$-ray spectrum gated by the $^8$Be channel.
Figure 5.9: Ratio between the populations of the ground state transition in $^{21}\text{Ne}$ (350 keV) and $^{22}\text{Ne}$ (1274 keV) for different energies of the coincident 2 $\alpha$-particles.

no variation should appear in the ratios given in Figs. 5.10 and 5.11. From both pictures one can see that the clusters are concentrated in the low energy region where the biggest deviation from a horizontal line is observed. In both cases the ratios go well below the expected multiple hit level (factor 10 to 100) indicating that the multiple hit events can be neglected in the $^8\text{Be}$ and $^{12}\text{C}$ spectra. This is confirmed by the study of the $\gamma$-decays.

With the observation of various channels like $2\alpha$, $3\alpha$, $^8\text{Be}$ or $^{12}\text{C}$ a systematic dependence on the excitation energy in the residual nucleus in cluster emission can be found. Note once more that in the systematics of Morgenstern et al. [Mor83], the average energy carried by individual nucleons and $\alpha$-particles is compared. Each individual nucleon carries approximately 16.4 MeV (four nucleons 66 MeV), one $\alpha$-particle carries approximately 22.3 MeV. Subtracting the $\alpha$-particle binding
Figure 5.10: Upper panel: the normalised total energy spectra ($E_{sum} = \Delta E + E$) as observed with the ISIS-charged particle detector system for the emission of 2 single $\alpha$’s, and of $^8$Be, in the reaction $^{18}$O+$^{13}$C. Lower panel: the ratio between these two experimental curves.
Figure 5.11: Upper panel: the total energy spectra \((E_{\text{sum}} = \Delta E + E)\) as observed with the ISIS-charged particle detector system for the emission of 3 single \(\alpha\)’s, and of \(^{12}\text{C}\), in the reaction \(^{28}\text{Si} + ^{24}\text{Mg}\). Lower panel: the ratio between these two normalised curves.
energy of 24 MeV, the 4-nucleons should have carried away 42 MeV, much more than the 22.3 MeV of the four bound nucleons. In addition, the unequal energies carried by single protons, and neutrons of 18.3 MeV and 13.2 MeV [Mor83], respectively, point to the influence of the Coulomb barrier, $E_{CB}$. The Coulomb barrier shifts the maximum energy expected for the emitted particle to larger values. Because of the linear dependence of the value of $E_{CB}$ on the emitted charge one can expect that the cluster emission energy spectra will be only slightly shifted to higher energies. The $E_{CB}$ for $^8$Be and 2 $\alpha$-particles for example has a difference of $\approx 2$ MeV (higher for $^8$Be). What one can see from the data is the opposite effect (see Figs. 5.5(lower panel) and 5.6(lower panel)): in both cases the maxima of the cluster emission energy spectrum is shifted to lower energies. One possible reason for this could be the difference coming from the kinematical variation of the energies (see Figs. 5.5(upper panel) and 5.6(upper panel)). Another possible explanation is given below.

The relative population of transitions following different decay modes, or alternatively, the ratio of the sequential emission to the binary cluster emission for a particular region in the decay scheme will be discussed. For the discussion the standard formulation for the energy spectra of evaporated particles can be used. The energy carried away in one evaporation process is given by Equation 5.16.

$$P(E) = E((2J + 1)/12)\sqrt{a}\exp\sqrt{4a(E - E_{rot})}. \tag{5.16}$$

Here, $J$ is the spin, $E_{rot}$ is the rotational energy and $a$ is the level density parameter. One of the clearest features of the sequential emission is the fact that the level density enters into each decay process, implying that in the sequential emission the decay phase space enters several times. Therefore, the sequential decay will dominate over the cluster decay. However, special effects like strong deformation or clustering in the parent nucleus may enhance the emission of larger fragments. Results concerning the population of deformed bands in $^{48}$Cr by the emission of $^8$Be from the compound nucleus have been studied with a similar procedure.
using the GASP-ISIS-detector set-up and have been described by Thummerer et al. [Thu01].

Another important feature of the chosen reaction channels is the selective population of states with natural parity in the residual nucleus, if only particles with spin zero, like the $^8$Be and $^{12}$C fragments, are emitted (see discussion on $^{40}$Ca parity doublets by Torilov et al. [Tor03]).

A pronounced difference in the excitation energy of the residual nucleus for the emission of clusters compared to the sequential processes has been observed. The relative yields are given (see Fig. 5.12) for the decay chains ($3\alpha$, $^8$Be+$\alpha$ or $^{12}$C), for the $^{28}$Si+$^{24}$Mg reaction. By inspecting the gated spectra one can see that the relative and absolute intensities of the lowest lying $\gamma$-ray transitions in $^{40}$Ca and $^{39}$K (the latter being indicative of a subsequent decay) are populated differently in the various particle gates. This is due to differences in the excitation energy and angular momentum in the residual nucleus $^{40}$Ca.

Figure 5.12: Intensity fractions for $\gamma$-ray transitions in residual nuclei gated by the $3\alpha$, ($^8$Be+$\alpha$) and $^{12}$C$^*(0^+_2)$ channels from the $^{28}$Si+$^{24}$Mg reaction.

Similarly, for the $^{13}$C+$^{18}$O reaction, the relative strengths of the subsequent neutron decays have been compared following the emission of $2\alpha$'s, or a $^8$Be, namely, the 0n, 1n and 2n channels, populating the residual nuclei $^{23}$Ne, $^{22}$Ne, and $^{21}$Ne respectively. As can be seen from Fig. 5.8 in the coincidence $\gamma$-ray spectra
with $^8\text{Be}$ the subsequent neutron emission is enhanced. Comparing the $\gamma$-ray spectra triggered on 2 $\alpha$-particles (Fold=2) the ratio between the population of $^{22}\text{Ne}$ (ground-state transition) in both matrices is $\approx 4$ and for $^{21}\text{Ne}$ (ground-state transition) only $\approx 1.5$. (The total ratio of events $N(2\alpha)/N(^{8}\text{Be}') \approx 1.5$).

**Gamma-ray coincidence spectra following the emission of $^6\text{Li}$ and $^7\text{Li}$**

In the second reaction ($^{18}\text{O}+^{13}\text{C}$) it can be seen (Fig. 5.13) that the calculated points of the energy loss of different fragments when flying through the silicon-telescopes are in good agreement with the experimental spectra. In Fig. 5.8 the $\gamma$-ray spectra gated by two $\alpha$-particles in different detectors and by two $\alpha$-particles coming in the same detector are shown. Looking at the spectra an interesting phenomenon was observed. Checking the $\gamma$-ray transitions which are observed in coincidence with the peak at 439 keV proves that the peak is the transition from the first excited state to the ground state in $^{23}\text{Na}$. From the charge conservation law it is not possible to have sodium (Na, $Z=11$) after the evaporation of 2 $\alpha$-particles from Si ($Z=14$). This experimental observation was carefully investigated and analysed. In the 2 $\alpha$-gated spectra (F=2) this peak (439 keV) is very weak (see Fig. 5.8), which is expected, after the suppression by the particle gate condition. In contrast, in the $^{8}\text{Be}'$ gate this peak is very strong and other peaks from the same nucleus ($^{23}\text{Na}$) are observable (see Fig. 5.8). This effect is presumably due to lithium emission, overlapping with the $^{8}\text{Be}$ particle gate. This hypothesis is tested below. In addition, the population of $^{21}\text{Ne}$ relative to $^{22}\text{Ne}$ increases when gating on $^{8}\text{Be}$ rather than $2\alpha$-particles (Fig. 5.9).

In order to investigate this the $^{8}\text{Be}'$ banana was cut in energy steps of 5 MeV and the $\gamma$-ray coincidences were analysed (see Fig. 5.14). One can see that $^{8}\text{Be}$ is stronger by a factor of $2 \rightarrow 2.5$ than the lithium. Further examining the ratios between the different neon-isotopes and sodium (see Fig. 5.15) one can see that the lithium is stronger at lower energies. Thus, looking at the energy spectra for 2 $\alpha$-particles registered in the same detector, a possible ‘scenario’ for the
cluster emission can be constructed: At lower energies presumably the emission of $^6$Li and $^7$Li is dominant, rather than mainly $^8$Be and some 2 uncorrelated (chance) $\alpha$-particles; in the highest energy region mainly 2 uncorrelated (double hit) $\alpha$-particle events are dominant.

### 5.2.3 Energy-to-angular momentum balance

By looking at the energy-to-angular momentum balance for the sequential emission processes versus the binary reactions the differences observed in the population of the final states in the residual nuclei ($^{23}$Ne and $^{40}$Ca) and the probability of a subsequent decay (because of a higher residual excitation energy) can be discussed further. The first step is to consider the angular momenta carried by, for example, 2 $\alpha$-particles or a $^8$Be cluster. The formula for the angular momentum,
Figure 5.14: Distribution of the populations of the ground-state transitions in $^{21}$Ne (350 keV), $^{22}$Ne (1274 keV) and $^{23}$Na (439 keV) for different energies of the coincidence gate ($^{18}$Be’ + double hit events).

$L_x$, of a fragment, $x$, for a given kinetic energy, $E_x$, is given by Equation 5.17.

$$L_x = kR\sqrt{1 - \frac{E_b}{E_{CM}}}$$  \hspace{1cm} (5.17)

where $E_b$ is the binding energy and the wave number $k$ is given by the product of mass times kinetic energy (as used in Equation 5.18). The $\alpha$-particle has mass, $m_1$, and an average energy, $E_1$. If the larger cluster consists of 2 $\alpha$-particles, the total mass is $2m_1$, and if the kinetic energy of the cluster is twice the energy of a single $\alpha$-particle emitted in the sequential process, there is no difference in the energy-to-angular momentum balance of the reaction according to Equation 5.17.
Figure 5.15: Ratio between the populations of the ground state transitions in $^{21}\text{Ne}$ (350 keV), $^{22}\text{Ne}$ (1274 keV) and $^{23}\text{Na}$ (439 keV) for different energies of the coincidence 2 $\alpha$-particles registered in the same detector. The curve is expected to give a horizontal line for pure double hit events (corresponding to the ratio between the probability of 1 and 2 neutron emission).

\[ k_{sBe} = \frac{\sqrt{4m_1^2E_1}}{\hbar} = 2k_{\alpha} \quad (5.18) \]

The origin of the difference in the population of the residual nucleus in a sequential compared to a one-step evaporation process for the same mass must be found in the emission process itself. There are two potential origins of the difference,

a) less angular momentum is carried away in the one-step process, because the geometry is more like a sticking situation, as in fission, or

b) in the sequential process, because of the dependence on the level density
(which enters two or three times for 2 or 3 \( \alpha \)-particles respectively) the emission of more total energy is emphasised.

### 5.3 Conclusions

The cluster fragments, \(^8\)Be or \(^{12}\)C carry away less energy from the compound nucleus than 2 or 3 sequentially emitted \( \alpha \)-particles and in the cluster scenario, corresponding to the same total (\(A,Z\)), the emission of another light charged particle from the residual nucleus is enhanced. The higher probability of sequential \( \alpha \)-particle emission than of cluster emission, is in good agreement with the statistical model. In terms of angular momentum, 2 \( \alpha \)-particles will have a higher total angular momentum because they carry more energy. Furthermore, in the sequential process the level densities emphasise the emission of higher total energy of the \( \alpha \)-particles.
Chapter 6

Summary

The work reported here is a result of experiments performed at the Laboratori Nazionali di Legnaro in Italy. Gamma-particle coincidence events were collected using the GASP germanium array in conjunction with the ISIS charged-particle detector. This combination enables discrimination between the different reaction channels by selecting the proper number of evaporated light charged-particles detected in the silicon telescopes.

Many new results have been obtained including properties of cluster emission and $\gamma$-ray spectroscopy of molecular states. In particular, several of the specific results are:

1) The $\gamma$-ray decay properties of neutron-rich isotopes of beryllium, close to the particle emission thresholds, have been studied. The $\gamma$-ray decay scheme has been extended for the levels up to the neutron decay threshold in $^{10}$Be. A new $\gamma$-ray transition from the isomeric $0^+_2$ state in $^{10}$Be was found and the population of the bandheads of the proposed rotational bands was measured.

2) The $\gamma$-ray decay of states populated in the $^{16}$O($^7$Li,2n), $^{16}$O($^7$Li,np) and $^{18}$O($^{13}$C,xn) reactions has been studied. The spectroscopy of $^{21}$Ne has been extended and interpreted following the concept of reflection asymmetric shapes due to octupole deformation. Two parity doublets with $K = 3/2$ and $1/2$ have been established and the corresponding quadrupole transitions between members of the
inversion doublets have been observed. The mirror nucleus to $^{21}$Ne, namely $^{21}$Na, was investigated and the behaviour of the Coulomb Energy Difference (CED) was interpreted.

Furthermore, the $\gamma$-ray decay properties of the neon isotopes $^{21,22,23}$Ne, populated in the $^{18}$O($^{13}$C,xn) reaction, where x=0,1,2, have been studied. New $\gamma$-ray transitions in $^{22}$Ne and $^{23}$Ne have been identified and the level schemes have been extended to higher spins. Possible spin assignments for the new transitions have been suggested. The decay scheme for the $K = 3/2$ band in $^{21}$Ne suggests a ‘pure’ compound nucleus reaction in which mainly yrast states are populated.

3) The emission of clusters, from the compound nucleus, and their energy spectra have been studied. The experimentally deduced ratio of uncorrelated $\alpha$-particles hitting the same silicon detector compared to multiple $\alpha$-particles from clusters is observed to be an order of magnitude smaller than expected. Moreover, the cluster fragments, $^8$Be or $^{12}$C, carry away less energy from the compound nucleus than 2 or 3 sequentially emitted $\alpha$-particles. Thus, the emission of another light particle from the residual nucleus after cluster emission is enhanced and a possible explanation is given.

These results will also enable future spectroscopic studies to be optimally designed for studying cluster states in light nuclei. Investigations are likely to continue to yield results on this interesting phenomenon of nuclear molecular states and cluster emission.
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