Effect of spin-spin interactions on polarisation observables from nucleon-nucleus scattering

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Overview

Optical model

Spin-spin interactions

Observable, polarisation transfer coefficient $D_{yy}$

Calculations of $D_{yy}$

Summary
Optical Model

For elastic scattering, the many-body problem arising from the complicated interaction between an incident particle and a nucleus can be represented by a complex 2-body potential.

Solving the Schrödinger equation with this complex potential yields predictions of basic observables.

For a spin-half particle scattering from a spin-zero nucleus the optical potential generally has the form:

\[ U(R) = U_{\text{cent}}(R) + U_{LS}(R) L \cdot S \]

Central term Spin-orbit term

This form of optical potential assumes the spin I of the target is zero.
In the new GSI/FAIR facility new short lived exotic nuclei with non-zero spin will be created.

Optical models which do not depend on $I$ may not be able to make reliable predictions of the observables which will be measured in experiments with these nuclei.

In order to determine accurate optical models for exotic nuclei with non-zero spin new types of potentials may be required.

Additional terms involving $I$ were first proposed by Feshbach \cite{1}, \cite{2}:

\[
S_{10}(\sigma, I) = \sigma \cdot I \\
S_{12}(\sigma, I, \hat{R}) = 3(\sigma \cdot \hat{R})(I \cdot \hat{R}) - \sigma \cdot I
\]

\begin{itemize}
  \item Central spin-spin term
  \item Tensor term
\end{itemize}

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Spin-Spin Interactions

Many attempts to describe spin-spin interactions using central, $S_{10}$ ($\sigma, I$) and second-rank tensor, $S_{12}$ ($\sigma, I, R$) terms \cite{3, 4, 5, 6}:

$$U(R) = U_{\text{cent}}(R) + U_{LS}(R) + U_{SS}(R)$$

$$U_{SS}(R) = U_{ss}^C(R)\sigma \cdot I + U_{ss}^T(R) \left[ 3(\sigma \cdot \hat{R})(I \cdot \hat{R}) - \sigma \cdot I \right]$$

Thesis work of McAbee \cite{7} described a generalised spin-spin operator:

$$S_{ik} = N_{ik} [I^{(i)} \otimes \sigma^{(k)}] \cdot [\hat{R}]^{(k)}$$

$$= N'_{ik} [I^{(i)} \otimes \sigma^{(k)}] \cdot C(\hat{R})^{(k)}$$

McAbee's calculation used a single valence nucleon model to derive folded potentials from effective nucleon-nucleon (NN) interactions.

Resulted in $S_{10}(\sigma, I)$ and $S_{12}(\sigma, I, R)$ terms, but also $S_{32}(\sigma, I, R)$ not previously discussed.

\cite{3} Davies & Satchler, Nucl. Phys. 53 (1964) 1 \quad \cite{4} A. P. Stamp, Phys. Rev. 153 (1967) 1052

\cite{5} C. J. Batty, Nucl. Phys. A178 (1971) 17 \quad \cite{6} A. H. Hussein & H. S. Sherif, Phys. Rev. C8 (1973) 518

\cite{7} T. L. McAbee, Ph. D. dissertation, University of North Carolina (1986)
Observable $D_{yy}$

Useful observable for investigating the existence of the spin-spin interaction is polarisation transfer coefficient $D_{yy}^{[8]}$:

$$D_{yy} = \frac{\text{Tr}(F\sigma_y F^\dagger \sigma_y)}{\text{Tr}(FF^\dagger)}$$

Where the direction of the y-axis is normal to the scattering plane.

$D_{yy}$ remains equal to one unless a term dependent on the orientation of the target spin, $\alpha$ is present in the potential.

It measures the extent to which the y-component of the incident proton polarisation is transferred to the outgoing proton polarisation.

For $D_{yy} = 1$, the y-component of the outgoing proton polarisation is the same as the y-component of the incident proton polarisation.

A scattering amplitude of the form:

\[ F(\theta) = A(\theta) + B(\theta)\sigma \cdot \hat{n} + \frac{C(\theta)}{\sqrt{I(I+1)}}\sigma \cdot I \]

gives a \( D_{yy} \) of:

\[ D_{yy} = \frac{\text{Tr}(F\sigma_y F^\dagger \sigma_y)}{\text{Tr}(FF^\dagger)} \]

\[ = 1 - \frac{4/3|C|^2}{|A|^2 + |B|^2 + |C|^2} \]

Expressing \( D_{yy} \) in this way shows:

- If \( C(\theta) = 0, D_{yy} = 1 \).
- If \( C(\theta) \neq 0, D_{yy} < 1 \).
- If \( C(\theta) \gg A(\theta) \) and \( C(\theta) \gg B(\theta) \), \( D_{yy} \to -1/3 \).
Measurements of $D_{yy}$ for 200 MeV protons elastically scattering from $^{10}\text{B}$ were carried out at Indiana University Cyclotron Facility (IUCF) \cite{Betker2005}.

Significant deviation from 1.

Effects of deformed ground state with spin parity $3^+$ in $^{10}\text{B}$ nucleus approximated using a coupled-channel calculation.

Failed to reproduce $D_{yy}$ at large angles.


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**D\textsubscript{yy} Calculation**

Using the standard optical potential from the IUCF calculation \cite{9} and a phenomenological central spin-spin potential from \cite{5} treated by DWBA gives a scattering amplitude of the form:

\[
F(\theta) = A(\theta) + B(\theta)\sigma \cdot \hat{n} - \frac{\mu}{2\pi\hbar^2} \langle \chi_{\mu_f}^-, IM_f | U_{SS}(R)\sigma \cdot I | \chi_{\mu_i}^+, IM_i \rangle
\]

\(U_{SS}(R)\) fitted to \(^9\)Be scattering data at 20 MeV.

Shows how this type of potential can result in a \(D_{yy}\) which deviates significantly from 1.

\begin{itemize}
  \item \cite{5} C. J. Batty, Nucl. Phys. A178 (1971) 17
  \item \cite{9} A. C. Betker et al., Phys. Rev. C 71 (2005) 2064607
\end{itemize}
Spin-Spin Calculation

Use effective NN interaction of the form:

\[ V_{NN}^{T=0,1} = V_{\text{cent}}^T + V_{\sigma\sigma}^T \sigma_1 \cdot \sigma_2 + V_{TR}^T S_{12} + V_{LS}^T L \cdot S \]

and a simple model for the $^{10}$B nucleus, assuming the valence p-shell proton and neutron are angular momentum coupled to give the spin $I = 3$ of the nucleus:

\[ \Psi_{10B} = u_p(r_1)u_n(r_2) \left[ Y_{1\lambda_1} \otimes \chi_{1/2\sigma_1} \right]_{3/2} \otimes \left[ Y_{1\lambda_2} \otimes \chi_{1/2\sigma_2} \right]_{3/2} \phi_{\text{core}} \]

Leads to a folding model potential:

\[ U(R) = \langle \Psi_{10B} | V_{op} + V_{on} | \Psi_{10B} \rangle \]
Spin-Spin Calculation

Folding of central $\sigma \cdot \sigma$ NN-interaction term leads to:

$$U_{\sigma \sigma}(R) = \langle \Psi_{10B}^0 | V_{\sigma \sigma}^{op} \sigma_p \cdot \sigma_n + V_{\sigma \sigma}^{on} \sigma_p \cdot \sigma_n | \Psi_{10B}^0 \rangle$$

$$= U_{\sigma \sigma}^{10}(R) \sigma \cdot I$$

$$+ U_{\sigma \sigma}^{12}(R) \left[ 3(\sigma \cdot \hat{R})(I \cdot \hat{R}) - \sigma \cdot I \right]$$

$$+ U_{\sigma \sigma}^{32}(R) S_{32}(\sigma, I, \hat{R})$$

Folding the tensor $S_{12}$ NN-interaction term leads to:

$$U_{TR}(R) = \langle \Psi_{10B}^0 | V_{TR}^{op} S_{op} + V_{TR}^{on} S_{on} | \Psi_{10B}^0 \rangle$$

$$= U_{TR}^{10}(R) \sigma \cdot I$$

$$+ U_{TR}^{12}(R) \left[ 3(\sigma \cdot \hat{R})(I \cdot \hat{R}) - \sigma \cdot I \right]$$

$$+ U_{TR}^{32}(R) S_{32}(\sigma, I, \hat{R})$$

$$+ U_{TR}^{34}(R) S_{34}(\sigma, I, \hat{R})$$
Focusing on central $\sigma \cdot \sigma$ contribution to the spin-spin $S_{10}$ or $\sigma \cdot I$ term:

$$F(\theta) = A(\theta) + B(\theta) \sigma \cdot \hat{n} - \frac{\mu}{2\pi\hbar^2} \langle \chi_{\mu \tau}^-, IM_f | V_{\sigma \sigma}^{10}(R) \sigma \cdot I | \chi_{\mu \tau}^+, IM_i \rangle$$

NN-interaction taken from $^{[10]}$ with exchange approximated by a zero-range pseudo-potential$^{[11]}$.

Similar structure to before but very small deviations in $D_{yy}$ from 1.

Need to include other terms:
- other contributions to $\sigma \cdot I$.
- and their exchange terms.
- higher rank terms, $S_{12}$, $S_{32}$ and $S_{34}(\sigma, I, r)$.

Summary

The spin-spin interaction $\mathbf{\sigma} \cdot \mathbf{I}$ is a sensible starting point for understanding the dependence of the optical potential on the target nucleus spin, $\mathbf{I}$. It can be used to calculate a $D_{yy}$ with significant deviation from 1.

However the $\mathbf{\sigma} \cdot \mathbf{I}$ interaction derived from the folding of the $\mathbf{\sigma} \cdot \mathbf{\sigma}$ term in the NN interaction alone is not sufficient to reproduce the experimental data.

Further work is required to include the other contributions to the $\mathbf{\sigma} \cdot \mathbf{I}$ term (from the tensor and spin-orbit (?) terms in the NN interaction), with a careful treatment of their exchange terms.

Also need to consider the contribution from the higher rank spin-spin interactions $S_{12} (\mathbf{\sigma}, \mathbf{I}, \mathbf{r})$, $S_{32} (\mathbf{\sigma}, \mathbf{I}, \mathbf{r})$ and $S_{34} (\mathbf{\sigma}, \mathbf{I}, \mathbf{r})$. 
Acknowledgements

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Thank you for listening...